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VIBRATION DYNAMICS OF A DEPENDENT SUSPENSION SYSTEM OF HALF-CAR MODEL WITH TIRE-ROAD SEPARATION

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Abstract: This paper presents a vibration analysis of a truck suspension system using a dependent vibrating half-car model to evaluate its dynamic behavior under tire-road contact and separation conditions. The proposed model consists of four degrees of freedom and considers the coupled motions between the front and rear wheels. A numerical approach is applied to calculate the vertical and roll responses of the suspension system under different road excitations. The equations of motion are formulated by incorporating the tire-road separation condition for both wheels, allowing the transition between contact and separation states to be examined. The effects of road profiles and suspension parameters on the dynamic responses are analyzed in both the time and frequency domains. The results show that considering tire-road separation provides a more realistic prediction of vehicle vibration behavior compared with the conventional assumption of continuous tire-road contact. The findings can be used as a reference for suspension design and for developing control methods aimed at improving vehicle stability and rollover resistance.

Keywords: Dependent Suspension, Vibration Dynamics, Tire-Road Separation, Roll Dynamics, Vehicle safety, Ride comfort.

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1. INTRODUCTION

The suspension system, which connects the vehicle body to its wheels, plays a crucial role in minimizing the effects of road-induced vibrations. By allowing relative motion between the wheels and the body through control arms or links, the suspension isolates the vehicle from road irregularities. Springs help shift the frequency of road disturbances into a more comfortable range for occupants, while dampers dissipate the energy generated by dynamic loads. Together, these components work to enhance ride comfort and maintain vehicle stability over uneven roads [1].

Rollover crashes remain one of the critical safety issues in vehicle operation, particularly for passenger cars and light trucks. According to the U.S. National Highway Traffic Safety Administration (NHTSA), approximately 10,000 fatalities were reported in rollover crashes in 2000, accounting for nearly one-third of occupant fatalities in passenger cars and light trucks [2]. The roll behavior of vehicles has been studied using full-car and half-car models. Although full-car models provide a more detailed representation of vehicle dynamics, their higher degrees of freedom increase the complexity of the analysis, especially when additional realistic factors are considered [3]. Under normal driving conditions, the roll angle of a vehicle is generally maintained within a small range of approximately 2-5 degrees. However, during cornering maneuvers or unstable operating conditions, the roll angle can increase to around 7-10 degrees depending on vehicle characteristics and driving conditions [4]. Ref. [5] reported that the suspension travel of light vehicles can reach approximately 0.25 m, which may contribute to a roll angle of up to 10 degrees.

Suspension systems can be broadly categorized into independent and dependent types. In independent suspension systems, each wheel operates separately, so movement in one does not influence the others. In contrast, dependent suspension systems involve a mechanical or fluid connection-such as a solid axle or interconnected hydraulic or pneumatic actuators-causing the movement of one wheel to affect the other [6, 7]. This study focuses on the dependent suspension system of a Huyndai HD 170 truck to analyze its vibration behavior, including roll dynamics.

In vehicle dynamics, it is commonly assumed that all wheels maintain continuous contact with the road surface [8]. However, in real-world conditions, tire-road separation can occur, as illustrated in Figure 1. This phenomenon typically occurs under high-frequency, high-amplitude road excitations or at high speeds [9]. When separation occurs, a gap forms between the tire and the road surface profile [10].

Tire-road separation refers to brief moments when the tires lose contact with the ground, with reconnection occurring only due to gravitational force [11]. This dynamic event compromises traction, handling, ride comfort, and increases the risk of rollover while the vehicle is in motion [12]. During such intervals, the tire is unable to deliver the required grip and traction, temporarily reducing vehicle control. This may result in sudden jolts or shifts in motion, reducing overall ride quality [13]. The loss and re-establishment of contact can transmit shocks and vibrations to passengers [14]. Since the friction between the tire and the road is crucial for braking, acceleration, and steering, any disruption in this contact can endanger vehicle safety [15, 16]. Therefore, analyzing the vehicle's vibration system is essential for understanding how tire-road separation impacts both ride comfort and safety.

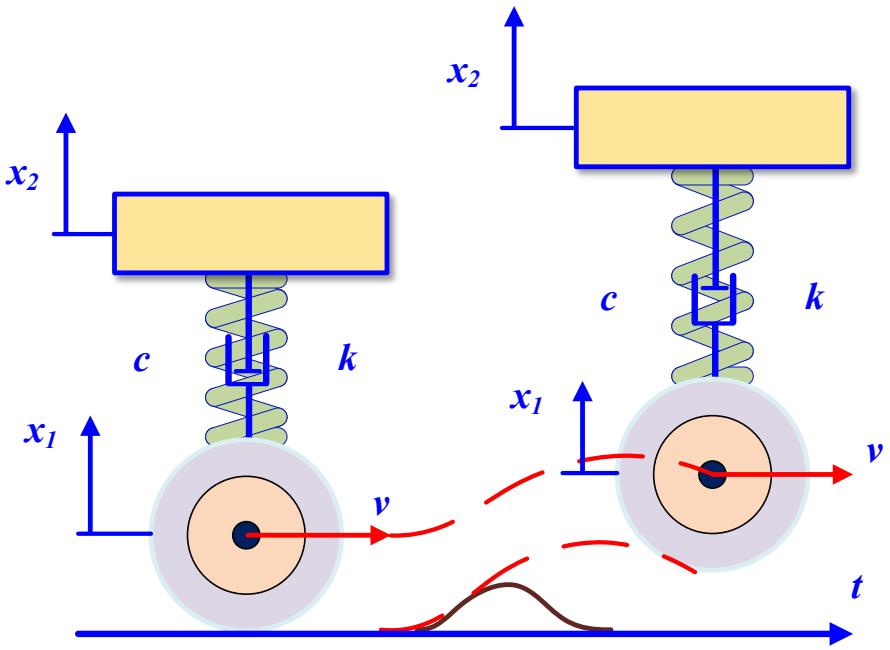


Figure 1. Wheel model and path during motion over a bumpy road.

The structure of the paper is as follows: Section 2 introduces the dependent suspension system through a half-car vibration model, including the governing equations and tire-road separation conditions. Section 3 presents simulation results for both vertical and roll responses under scenarios with and without tire-road separation. Finally, the conclusions highlight the main findings of the research.

2. VEHICLE MODELING

In this section, a vibrating half-car model of a truck is presented along with its corresponding system of differential equations. Additionally, the equations have been extended to account for the no-contact assumption based on the introduced tire-road separation condition.

2.1. Governing equation system

The most fundamental approach to analyzing and improving lateral stability is by using the half-car model. Figure 2 illustrates the vibratory system with 4 DOF as seen from the front of the Hyundai 170D truck. This type of suspension functions as a solid connection between the two wheels, transferring movement and force from one wheel to the other. It is commonly referred to as a dependent suspension system. To make the vibration analysis simpler, this study ignores the damping effect of the tire. This is because tire damping is small compared to the main damping in the system. Leaving it out allows the analysis to focus on the key damping sources and better understand the vibration behavior.

In dynamic analysis, the suspension may be assumed to consist of rigid bodies connected by springs and dampers. An upper rigid body representing the car body is presented, with a mass of m_2, referred to as the sprung mass. The bar has a inertia moment of I_{x2} and represents the front mass of vehicle body. A lower rigid body represents the unsprung mass, with a mass of m_1 and a moment of inertia of I_{x1}. The stiffness of the tire is symboled by k_t. The truck's

suspension system includes a damping coefficient c and a stiffness coefficient k. It is typically designed with symmetrical suspensions on the left and right sides, meaning both the damping and stiffness values are equal for each side. The roll angles are represented φ_1, φ_2, and excitation profiles y_1, y_2.

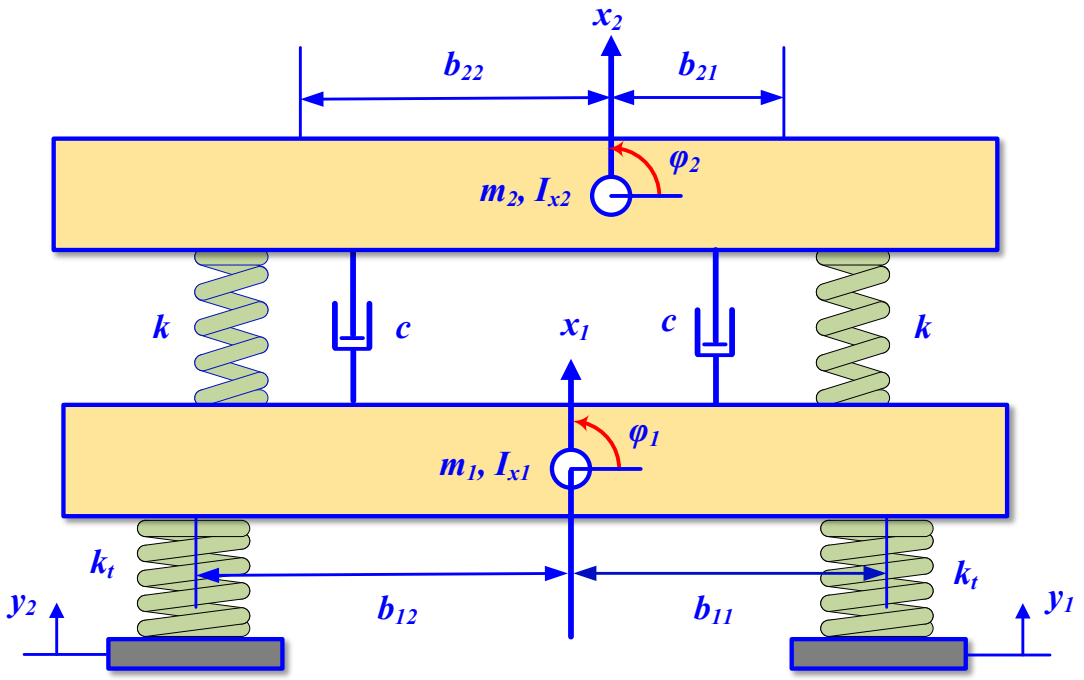


Figure 2. Half-car model of a dependent suspension system.

By applying Newton's second law, the vibration dynamics in the in-contact state are described by the following equations:

$$m_2\ddot{x}_2 + k(x_2 - x_1 + b_{21}\varphi_2 - b_{21}\varphi_1) + k(x_2 - x_1 - b_{22}\varphi_2 + b_{22}\varphi_1) + c(\dot{x}_2 - \dot{x}_1 + b_{21}\dot{\varphi}_2 - b_{21}\dot{\varphi}_1) + c(\dot{x}_2 - \dot{x}_1 - b_{22}\dot{\varphi}_2 + b_{22}\dot{\varphi}_1) = 0 \quad (1)$$

$$I_{x2}\ddot{\varphi}_2 + b_{21}c(\dot{x}_2 - \dot{x}_1 + b_{21}\dot{\varphi}_2 - b_{21}\dot{\varphi}_1) + b_{21}k(x_2 - x_1 + b_{21}\varphi_2 - b_{21}\varphi_1) - b_{22}c(\dot{x}_2 - \dot{x}_1 - b_{22}\dot{\varphi}_2 + b_{22}\dot{\varphi}_1) - b_{22}k(x_2 - x_1 - b_{22}\varphi_2 + b_{22}\varphi_1) = 0 \quad (2)$$

$$m_1\ddot{x}_1 + k_t(x_1 - y_1 + b_{11}\varphi_1) + k_t(x_1 - y_2 - b_{12}\varphi_1) - k(x_2 - x_1 + b_{21}\varphi_2 - b_{21}\varphi_1) - c(\dot{x}_2 - \dot{x}_1 + b_{21}\dot{\varphi}_2 - b_{21}\dot{\varphi}_1) - k(x_2 - x_1 - b_{22}\varphi_2 + b_{22}\varphi_1) - c(\dot{x}_2 - \dot{x}_1 - b_{22}\dot{\varphi}_2 + b_{22}\dot{\varphi}_1) = 0 \quad (3)$$

$$I_{x1}\ddot{\varphi}_1 + b_{11}k_t(x_1 - y_1 + b_{11}\varphi_1) - b_{12}k_t(x_1 - y_2 - b_{12}\varphi_1) - b_{21}k(x_2 - x_1 + b_{21}\varphi_2 - b_{21}\varphi_1) - b_{21}c(\dot{x}_2 - \dot{x}_1 + b_{21}\dot{\varphi}_2 - b_{21}\dot{\varphi}_1) + b_{22}k(x_2 - x_1 - b_{22}\varphi_2 + b_{22}\varphi_1) + b_{22}c(\dot{x}_2 - \dot{x}_1 - b_{22}\dot{\varphi}_2 + b_{22}\dot{\varphi}_1) = 0 \quad (4)$$

The half-car vibration model is excited by the rough surfaces. The amplitudes of road way are represented by Y_1, Y_2 and $\omega_1; \omega_2$ denote the angular frequencies. It is noted that when the values of the two angular frequencies are equal, it is symboled by ω. The road profiles are functions of time, expressed as: $y_1 = Y_1.\sin(\omega_1.t)$ và $y_2 = Y_2.\sin(\omega_2.t)$. When the wheels lose contact with the road under severe conditions, the elastic force from the tire acting on the road and the reaction force from the road become zero, and only the gravitational force remains acting on the system. The displacement and roll responses of the body still follow equations (1) and (2), while the motion equations of the unsprung mass are given by:

$$m_1\ddot{x}_1 + (m_1 + m_2)g - k(x_2 - x_1 + b_{21}\varphi_2 - b_{21}\varphi_1) - c(\dot{x}_2 - \dot{x}_1 + b_{21}\dot{\varphi}_2 - b_{21}\dot{\varphi}_1)$$
$$-k(x_2 - x_1 - b_{22}\varphi_2 + b_{22}\varphi_1) - c(\dot{x}_2 - \dot{x}_1 - b_{22}\dot{\varphi}_2 + b_{22}\dot{\varphi}_1) = 0 \quad (5)$$

$$I_{x1}\ddot{\varphi}_1 - b_{21}c(\dot{x}_2 - \dot{x}_1 + b_{21}\dot{\varphi}_2 - b_{21}\dot{\varphi}_1) - b_{21}k(x_2 - x_1 + b_{21}\varphi_2 - b_{21}\varphi_1)$$
$$+b_{22}c(\dot{x}_2 - \dot{x}_1 - b_{22}\dot{\varphi}_2 + b_{22}\dot{\varphi}_1) + b_{22}k(x_2 - x_1 - b_{22}\varphi_2 + b_{22}\varphi_1) = 0 \quad (6)$$

2.2. The tire-road separation condition

Depending on the tire's static compression, the traveling wheel can exist in two states. If the vertical displacement of the tire is less than its static compression, it remains in contact with the ground. However, if the vertical movement exceeds the static compression, the tire loses contact and enters a free-flight state. Combining the vertical displacement with the torsional motion of the unsprung mass creates the displacement of the wheels. The relative displacement of the left wheel is calculated as $x_{1l} = x_1 + b_{11}\varphi_1 - y_1$, while for the right wheel it is $x_{1r} = x_1 + b_{12}\varphi_1 - y_2$. Therefore, the tire-road separation condition is given by:

In-contact condition for left wheel: $x_1 + b_{11}\varphi_1 - y_1 < x_{T1}$ (7)

No-contact condition for left wheel: $x_1 + b_{11}\varphi_1 - y_1 \geq x_{T1}$ (8)

In-contact condition for right wheel: $x_1 + b_{12}\varphi_1 - y_2 < x_{T2}$ (9)

No-contact condition for right wheel: $x_1 + b_{12}\varphi_1 - y_2 \geq x_{T2}$ (10)

The condition for the left wheel to lose contact with the road surface is given by equation (8), while the separation condition for the right wheel is defined by equation (10). Therefore, three possible separation scenarios can occur. When both equation (8) and equation (10) are satisfied, the left and right wheels separate from the road surface simultaneously. If only equation (8) together with equation (9) is satisfied, only the left wheel loses contact with the road surface. Conversely, when equation (7) and equation (10) are satisfied, only the right wheel separates from the road surface.

Where, the static compressions for the two tires are:

$$x_{T1} = \frac{\left(m_1 \frac{b_{12}}{b_{11}+b_{12}} + m_2 \frac{b_{22}}{b_{21}+b_{22}} \right) g}{k_t} \text{ and } x_{T2} = \frac{\left(m_1 \frac{b_{11}}{b_{11}+b_{12}} + m_2 \frac{b_{21}}{b_{21}+b_{22}} \right) g}{k_t} \quad (11)$$

The governing equations of the proposed model are solved numerically using MATLAB ode45 under the specified tire-road separation condition. The ode45 solver, based on an explicit Runge-Kutta method, is applied to obtain the time response of the suspension system. The calculated responses are then used to evaluate the dynamic behavior and determine the occurrence of tire-road separation under different excitation conditions.

3. RESULTS AND DISCUSSION

The vibration dynamics of a practical vehicle will be thoroughly analyzed using the input parameters listed in Table 1. These parameters represent realistic physical and mechanical properties relevant to the vehicle's suspension and structural system. By incorporating actual data, the study aims to ensure that the dynamic analysis closely reflects real-world behavior.

Table 1. The parameters of a half-car model of Hyundai HD170.

Parameter	Value [Unit]	Parameter	Value [Unit]
m_2	6,300 *kg*	I_{x2}	2,187 $kg.m^2$
m_1	700 *kg*	I_{x1}	211 $kg.m^2$
b_{11}	1 *m*	b_{12}	1.04 *m*
b_{21}	0.93 *m*	b_{22}	0.97 *m*
k	390,000 *N/m*	k_t	1,500,000 *N/m*
c	25,000 *Ns/m*		

The time response is commonly used to characterize the dynamic behavior of a system. In the context of the vibrating model, the time response illustrates how the vertical displacement and roll motion evolve over the specified time interval. By analyzing these responses, one can gain insights into the system's stability, transient behavior, and steady-state performance. This information is particularly valuable for evaluating ride comfort and handling characteristics in practical vehicle applications.

In Figure 3, the time responses of the dependent suspension at a low input frequency $\omega = 7\,rad/s$ are illustrated. Due to the low road excitation frequency, the system primarily responds within the contact period, and the values of the separation indicators I_1, I_2 remain zero throughout. It is important to note that the road excitations on both sides are identical, causing the displacements of both wheels x_{1r}, x_{1l} to coincide with each other. The response of both wheels follows the shape of the road excitation; however, the amplitudes reach approximately 0.12m,

which is higher than the amplitude of the road input $Y_1 = 0.1m, Y_2 = 0.1m$. Therefore, the roll angle of the body is effectively zero, while the vertical displacement of the truck body φ_2 vibrates harmonically with an amplitude of approximately 0.17 m.

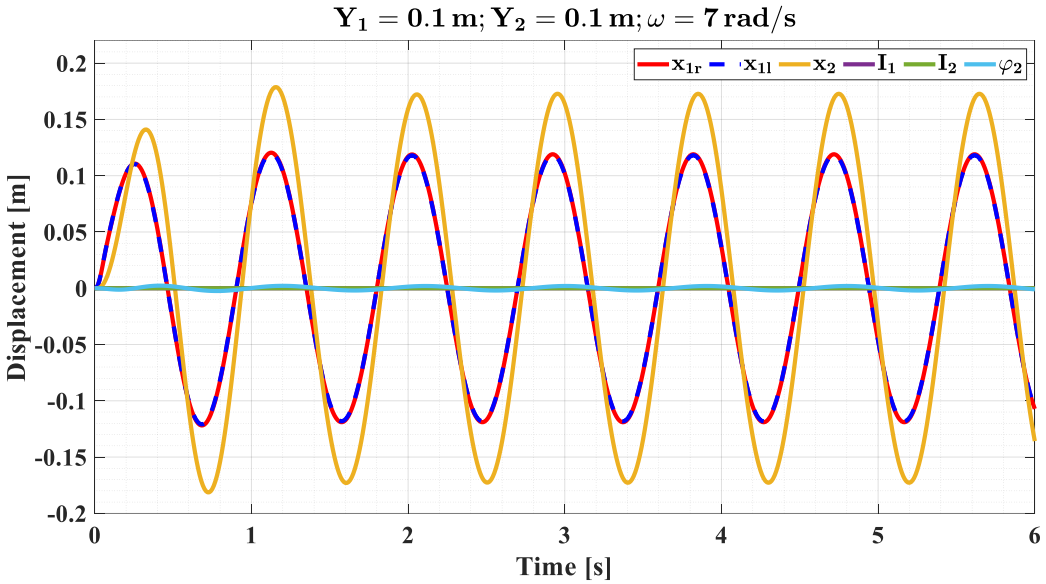


Figure 3. Time responses in the contact state.

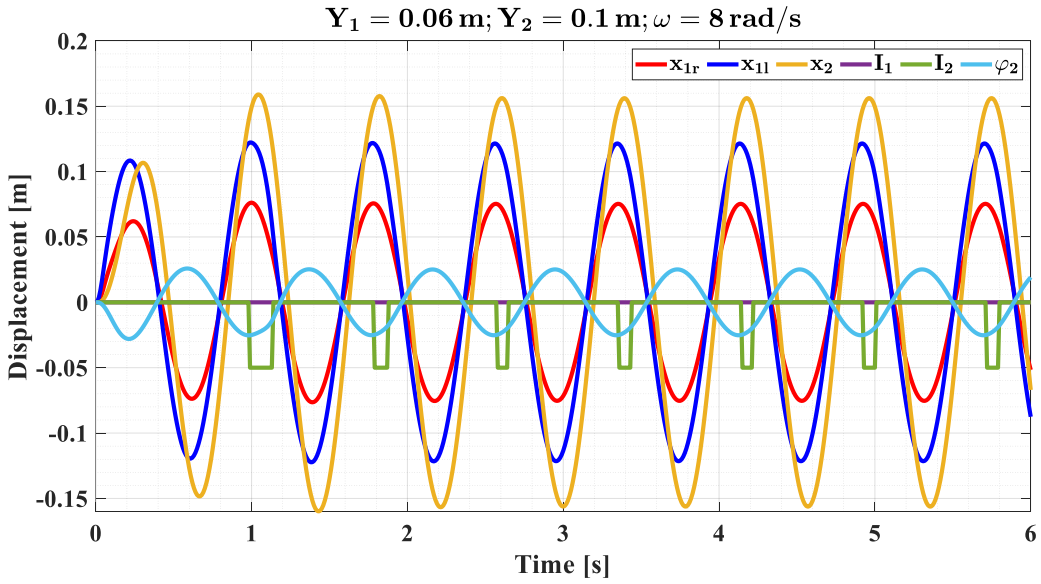


Figure 4. Time responses with right wheel separated.

To observe when a wheel loses contact with the ground, the input data is kept the same while the two road excitations are changed. The input frequency is increased $\omega = 8\ rad/s$, and the left wheel is given a new road amplitude $Y_1 = 0.06m$. As shown in Figure 4, the right wheel x_{1r} shows a smaller response. As a result, the vertical vibration of the vehicle body decreases to 0.16 m. Interestingly, in this case, the left wheel becomes separated from the road. This

separation is detected using an indicator function I_2, where each separation period is marked by a value of 0.05 or -0.05. Meanwhile, the right wheel stays in contact with the road, shown by an indicator I_1 value of 0. The roll motion of the vehicle body φ_2 now vibrates in a harmonic pattern with an amplitude of 0.02 radians. The amplitude of the roll motion is approximately equal to 1^0, which is at the limit of vehicle stability.

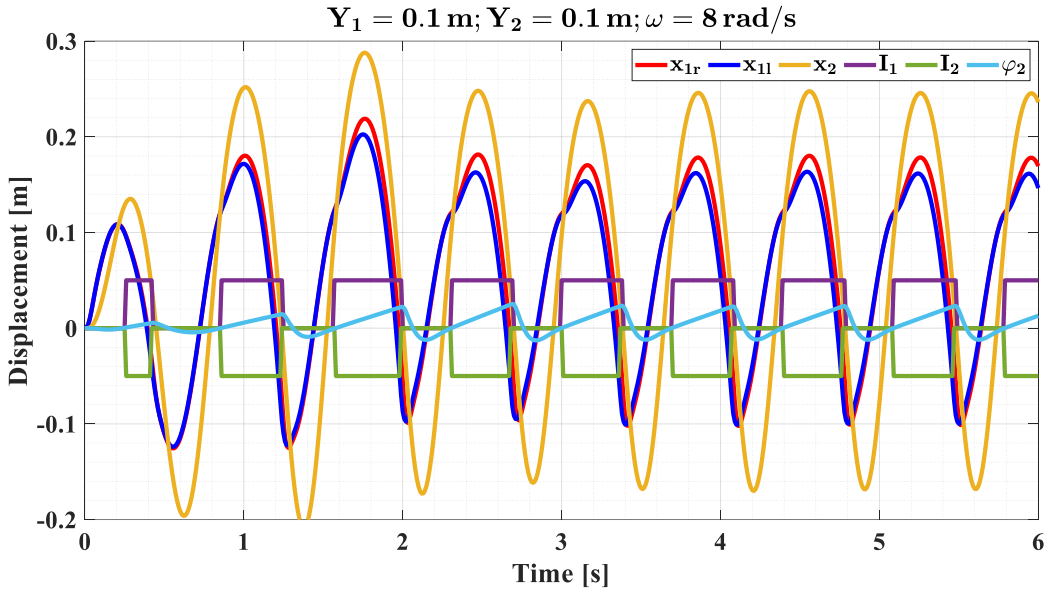


Figure 5. Time responses with both wheels separated.

Building on the input data from Figure 3, the frequency of the road excitations is increased to $\omega = 8\ rad/s$. As a result, both wheels enter a free-fall state, as shown in Figure 5, with the separation indicators reaching values of 0.05 and -0.05, respectively. Both wheels are separated, leading to an increased displacement of the truck body, reaching up to 0.25 m after the transient period. Even though both wheels lose contact with the road, their dynamic responses remain closely aligned. This leads to a roll angle so small that it can be considered negligible. Therefore, in terms of roll dynamics, it can be inferred that having both wheels separated at the same time may actually result in better stability than having only one wheel separated.

Another way to analyze vibration dynamics is by using a phase portrait, which shows both displacement and velocity at each simulation point. In the case of wheel separation, Figure 6 illustrates the phase portraits of the vertical responses for both tires in three-dimensional space, showing displacement $x_{1r}\,(x_{1l})$ and vertical velocity $v_{1r}\,(v_{1l})$ during the two-wheel hop over time. These 3D curves are generated under separation conditions with high vibration amplitude $Y_1 = 0.1m, Y_2 = 0.1m$ and low input frequency $\omega = 9\,rad/s$. In this scenario, the relationship between displacement and velocity forms a helical pattern, indicating chaotic system behavior. When projected onto 2D planes, the phase curves of the half-car model do not show harmonic responses-this behavior appears only under separation conditions. If both tires remain in contact with the road, the system behaves more regularly.

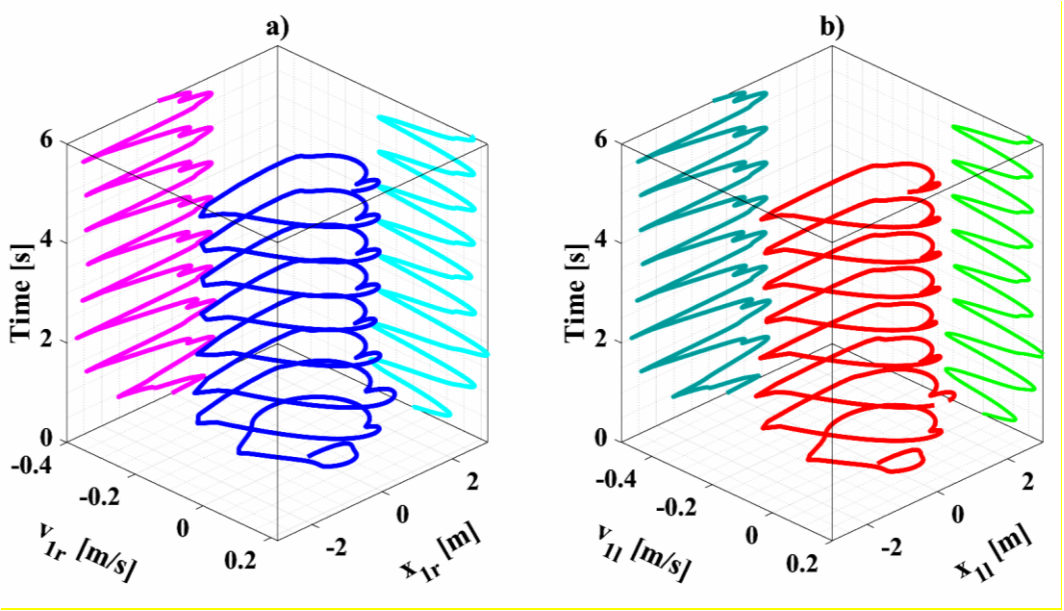


Figure 6. Phase portraits of wheel's motion.

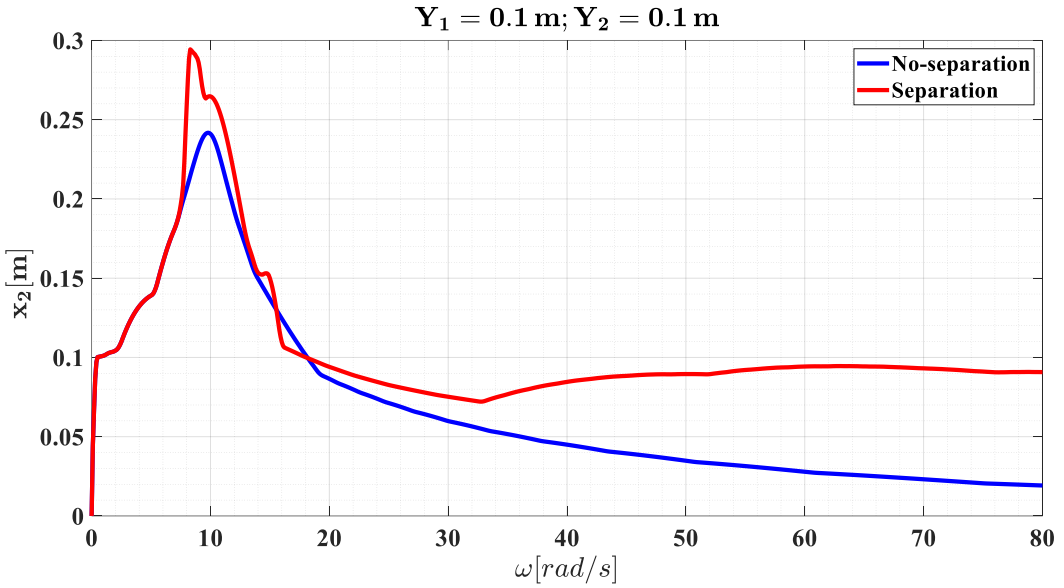


Figure 7. Frequency response of the vehicle body.

These previous graphs are plotted only at a specific input frequency. By capturing the amplitude from each graph, we can plot the system's frequency response. The frequency domain allows for the simulation of vibration dynamics across a wide range of input data. The frequency domain of half-car model of the truck is depicted in Figure 7 over input frequency of road excitation from 0 to $\omega = 80\,rad/s$. Furthermore, the previous time responses do not provide a comparison between the separation and no-separation cases.

When the tire-road separation assumption is applied, the actual response deviates significantly from the no-separation curve. In this case, the response with separation is generally higher than that without separation. This highlights the importance of accounting for separation effects in dynamic analysis, as neglecting them can lead to underestimating vibration

amplitudes and misjudging vehicle performance. Including the separation assumption allows for more accurate modeling of real-world behavior, especially under extreme road conditions or high-speed excitation. The two curves have the same resonant frequency. The first resonant frequency of the truck body is estimated to be at $\omega = 9 rad/s$, while the second resonant frequency is not clearly identifiable.

To gain a deeper understanding of the system's behavior, the study examines the frequency response of the unsprung mass, as shown in Figure 8. Before reaching the resonant zone, both the separation and no-separation curves coincide completely. Within the resonant zone, where the input frequency ranges from 8 to 12 rad/s, the separation curve becomes significantly higher than the no-separation curve, reaching an amplitude of over 0.2 m-more than twice the road input amplitude. However, beyond the first resonant zone, the two curves become nearly identical. The first resonant frequency is mainly affected by the sprung mass, while the second resonant frequency is mainly influenced by the unsprung mass. The two resonant zones generate dynamic vertical forces transmitted to the passengers and the road surface. Therefore, accurate simulation of the vehicle body frequencies is important for evaluating vehicle vibration characteristics. When the excitation frequency is close to these resonant frequencies, the vibration amplitude can increase significantly. This may reduce ride comfort and increase the dynamic tire load acting on the road.

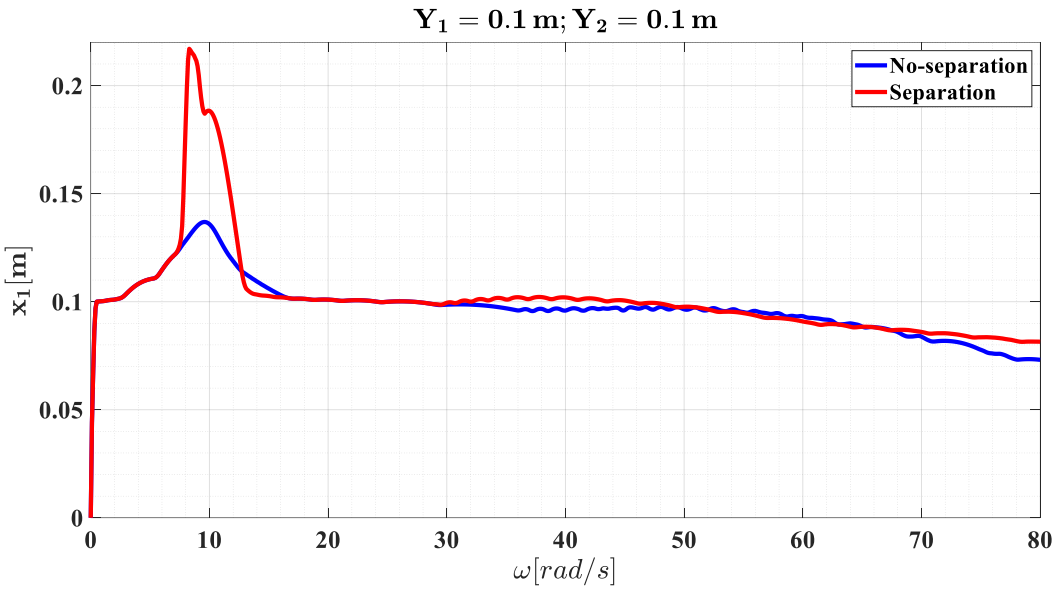


Figure 8. Frequency response of the unsprung mass.

The roll response is one of the most uncomfortable vibrations experienced in a vehicle. Figure 9 presents a comparison of the roll frequencies of the unsprung mass under both contact and no-contact conditions. If tire-road separation is ignored, the roll motion appears minimal, as shown by the no-separation curve. However, when the tire-road separation is considered, the roll angle in the real system becomes significant, especially at the first and second resonant zones around 9 rad/s and 34 rad/s, respectively. In these zones, the roll angles reach approximately 6^0 and 2^0, values that can significantly reduce ride comfort for both the goods and the driver. Investigating roll dynamics is crucial because excessive roll can affect vehicle stability and passenger comfort. Understanding these dynamics helps in designing suspension systems that minimize harmful vibrations and improve overall safety.

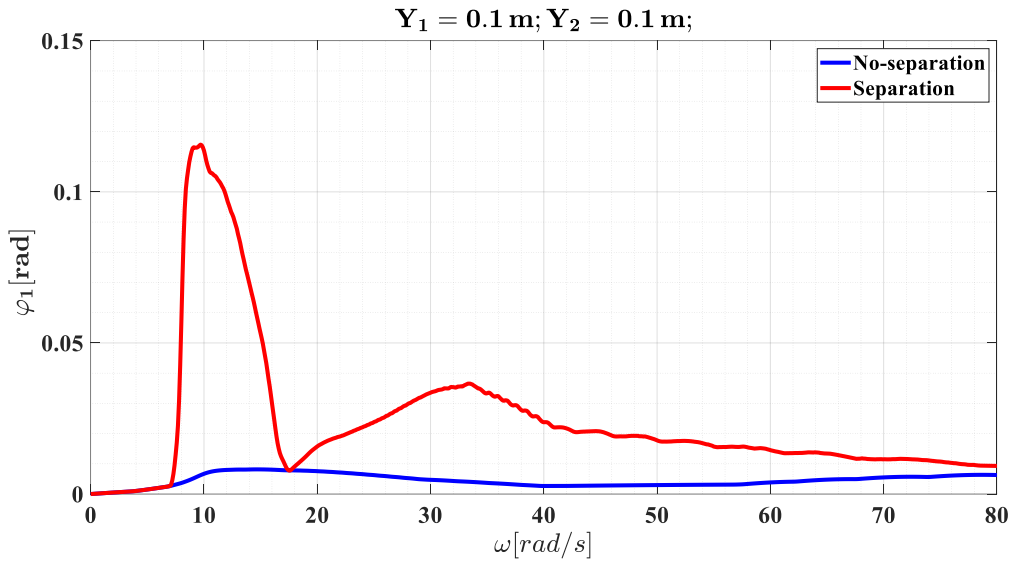


Figure 9. Roll motion of unsprung mass.

The dynamic tire forces generated from road excitation are transmitted to the driver and cargo through the suspension system and truck body. Therefore, evaluating the roll dynamics of the vehicle body is important for assessing vehicle stability. As shown in Figure 10, the roll angle of the rigid body is compared between the tire-road separation and continuous contact conditions. The frequency response can be divided into three main regions. The two resonance regions are related to the coupled vibration characteristics of the sprung and unsprung masses. In these regions, the roll angle under the separation condition is generally higher than that under the contact condition. However, an opposite trend is observed in the intermediate frequency range between the two resonance regions, particularly from 16 rad/s to 26 rad/s. The maximum roll angle reaches approximately 0.13 rad (7.45^0), indicating a considerable influence on vehicle stability. Such a large roll response may increase the risk of unsafe operating conditions, especially during severe road excitation or cornering maneuvers. Therefore, accurate modeling of tire-road separation is necessary, since simplified assumptions may underestimate the roll response and fail to capture critical stability characteristics.

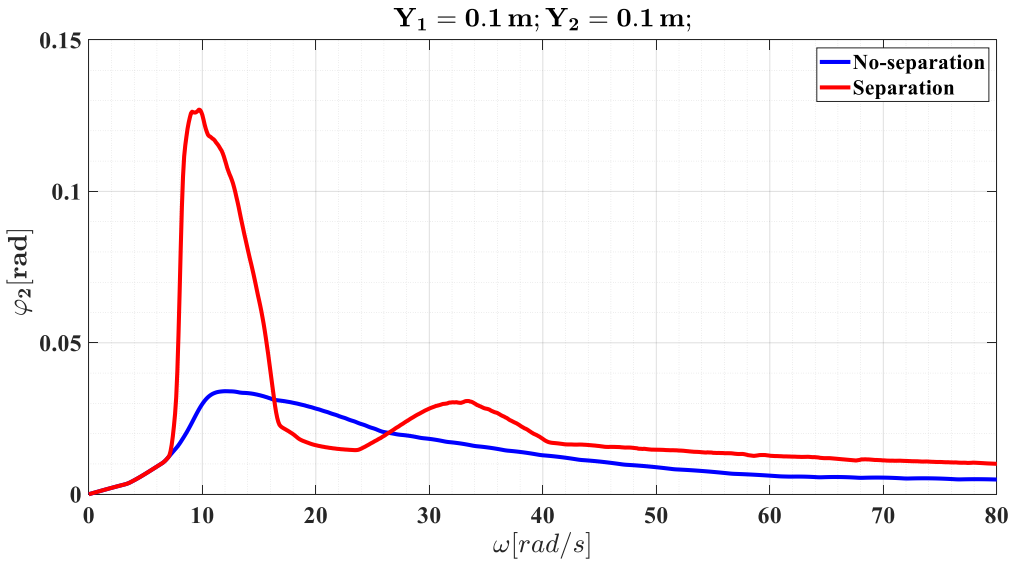


Figure 10. Roll motion of vehicle body.

4. CONCLUSIONS

This study presents a vibration analysis of a truck suspension system with consideration of both vertical and roll motions. The tire-road separation condition is incorporated into the half-car model to investigate the vehicle responses under different road excitations. The results obtained in the time and frequency domains show that neglecting tire-road separation may lead to an inaccurate prediction of roll angle and vertical displacement, especially near resonance regions. This difference indicates that the continuous contact assumption may underestimate potential safety issues related to excessive roll motion and rollover risk. The results also show that the separation of both wheels can produce different roll responses compared with the case of single-wheel separation, depending on the excitation conditions. Therefore, accurate modeling of tire-road contact loss is necessary for evaluating vehicle stability and improving suspension design and control strategies. In the next step, the research will use the ADAMS environment to validate the analytical results. The simulation results obtained from ADAMS will be compared with the proposed theoretical model to evaluate its accuracy and reliability.

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REFERENCES

[1]. Trzesniowski, M., 2023. Suspension system. Springer Vieweg. https://link.springer.com/book/10.1007/978-3-658-39847-7

[2]. National Highway Traffic Safety Administration, “Characteristics of fatal rollover crashes”, DOT HS 809 438, Washington, DC, Apr. 2002. https://crashstats.nhtsa.dot.gov/Api/Public/ViewPublication/809438

[3]. R. N. Jazar and H. Marzbani, *Vehicle Vibrations: Linear and Nonlinear Analysis, Optimization, and Design*. Cham, Switzerland: Springer, 2024. https://link.springer.com/book/10.1007/978-3-031-43486-0

[4]. Parczewski, K., & Wnęk, H. (2017). The influence of vehicle body roll angle on the motion stability and maneuverability of the vehicle. Combustion Engines, 56(1), 133-139. https://www.mendeley.com/catalogue/9fb1f04a-3849-3b88-acdf-a4d235920eb1/

[5]. J. Wang and S. Shen, “Integrated vehicle ride and roll control via active suspensions”, Vehicle System Dynamics, vol. 46, pp. 495-508, Sep.2008. https://www.tandfonline.com/doi/full/10.1080/00423110801993128

[6]. N. Rosam and J. Darling, “Development and simulation of a novel roll control system for the interconnected hydragas suspension”, Vehicle System Dynamics, 27, pp. 1-18, 1997. https://www.tandfonline.com/doi/abs/10.1080/00423119708969320

[7]. Wang, L., Zhang, N. and Du, H., 2009, December. Design and experimental investigation of demand dependent active suspension for vehicle rollover control. In Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference (pp. 5158-5163). IEEE. https://ieeexplore.ieee.org/document/5399853

[8]. N. Jazar, R. (2013). Advanced vibration: A modern approach. Springer Science & Business Media. https://link.springer.com/book/10.1007/978-1-4614-4160-1

[9]. ˇZuraulis, V., Levulyt˙e, L., & Sokolovskij, E. (2014). The impact of road roughness on the duration of contact between a vehicle wheel and road surface. Transport, 29 (4), 431–439. https://www.tandfonline.com/doi/abs/10.3846/16484142.2014.984330

[10]. Nguyen, Q. D., Jazar, R.N. (2022). Numerical Simulation and Experimental validation of the Tire-Road Separation in Quarter-car Model. 16th International Conference on Automotive and Mechanical Engineering ICAME. https://publications.waset.org/10013266/numerical-simulation-and-experimental-validation-of-the-tire-road-separation-in-quarter-car-model

[11]. Khazaie, A., Hussaini, N., Marzbani, H. and Jazar, R.N., 2018. Quarter Car Suspension Model with Provision for Loss of Contact with the Road. In Nonlinear Approaches in Engineering Applications (pp. 167-208). Springer. https://link.springer.com/chapter/10.1007/978-3-319-69480-1_7

[12]. J. Y. Wong, *Theory of Ground Vehicles*. Hoboken. John Wiley & Sons, 2022. https://onlinelibrary.wiley.com/doi/book/10.1002/9781119719984

[13]. Nguyen, Q. D., Milani, S., Vo, D., Marzbani, H., & Jazar, R. N. (2021). Vibration analysis of the bicycle-car model considering tire-road separation. SAE International Journal of Commercial Vehicles (02-15-02-0004). https://doi.org/10.4271/02-19-04-0021

[14]. Nguyen, Q. D., Milani, S., Marzbani, H., & Jazar, R. N. (2021a). Tire-road separation time reduction by an adaptive pid controller utilizing particle swarm optimization algorithm. SAE International Journal of Commercial Vehicles, 14 (02-14-04-0033). https://www.sae.org/publications/technical-papers/content/02-14-04-0033/

[15]. Nguyen, Q. D., Milani, S., Marzbani, H., & Jazar, R. N. (2021b). Vehicle vibrations analysis of the quarter-car model considering tire-road separation. In Dai l., Jazar r. (eds) nonlinear approaches in engineering applications: Design engineering problems. Springer, New York. https://link.springer.com/chapter/10.1007/978-3-030-82719-9_9

[16]. Nguyen, Q. D., Milani, S., Marzbani, H., & Jazar, R. N. (2022). Vehicle ride analysis considering tire-road separation. journal of Sound and Vibration, 521, p.116674. https://www.sciencedirect.com/science/article/abs/pii/S0022460X21006854