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AN OPTIMIZATION-BASED APPROACH FOR STATIC ANALYSIS OF MULTI-SEGMENT MOORING LINES WITH CLUMP WEIGHTS

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Abstract. Static configuration analysis of multi-segment mooring systems equipped with clump weights poses a significant computational challenge due to the high non-linearity and sensitivity of the governing equations. This study proposes an optimization-based approach to determine the equilibrium shape of mooring lines, bypassing the limitations of traditional trial-and-error or simplified analytical methods. By incorporating the fundamental catenary equations with force balance and displacement continuity conditions into the objective function, the problem is reformulated as a search for the optimal inclination angle at the lower point of each segment. Two metaheuristic algorithms, Particle Swarm Optimization (PSO) and Genetic Algorithm (GA), are implemented to solve the system of equations. Results demonstrate that the proposed method achieves high accuracy when validated against Finite Element Analysis (FEA), yielding negligible discrepancies (maximum error of 0.3% for inclination angles and 0.0% for mooring tensions). The study highlights that PSO exhibits superior convergence speed and stability compared to GA, particularly in complex configurations involving multiple clump weights.

Keywords: Mooring line, Floating structure, Calculation method, Optimization, Particle Swarm Optimization (PSO) and Genetic Algorithm (GA).

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1. INTRODUCTION

The mooring system, particularly the mooring line, plays a vital role in a floating structure to keep the floating body stable under the effect of the environment and the vessel. The popular mooring system used can be a single-point mooring (SPM) or a spread mooring system with multiple lines. The mooring lines may be arranged in either a catenary configuration (slack mooring) or a taut-leg configuration (taut mooring) [1, 2].

The clump weights are defined as additional weights attached to the mooring lines so that the mooring performance can be enhanced by increasing the restoring force of the mooring system [3]. Thus, in the case of a narrow plan or the offset of the floating body needs to be limited, clump weight is a good solution. Currently, the behaviour of the mooring line is investigated using two approaches, including analytical and numerical ones.

In 1976, Bertaux proposed and summarized the theoretical foundation for mooring line analysis [2]. The author elaborated on the fundamental equations governing the catenary profile of the mooring line, along with key quantities such as line tension, floating body offset, and anchor or sinker uplift force. However, in this analytical approach, these equations cannot be solved without simplifications. In particular, mooring lines with clump weights present additional analytical difficulties. At least one of the parameters-such as the mooring line angle, the floating body offset, or the water depth-must be specified to obtain a solution to the governing equations.

Currently, Finite Element Analysis (FEA) is the most popular approach to study the behaviour of the mooring line. In this approach, open-source or commercial software packages - such as OpenFAST, NREL FAST, CHARM3D, ANSYS AQWA, and OrcaFlex - can be utilized for the analysis [1, 4-8]. These tools are used to analyze a wide range of floating structures, such as floating storage and offloading units (FPSOs), floating wind turbines (FWTs), floating docks, and mooring buoys. These tools are capable of modeling a wide range of scenarios, including static and dynamic loading conditions, as well as various mooring configurations-such as catenary, single-line, and multi-line systems with or without sinkers. In our previous work [9], Sap2000 has been proposed to model the mooring line. Results show that this software can simulate a mooring line by a cable element using a nonlinear solver.

In addition, since the early 2000s, studies have focused on optimizing mooring line weight and the horizontal offset of the floating body. In 2007, Shafieefar et al. [10] proposed a procedure for the optimization of the mooring design of floating platforms using GA algorithm. The objective function is to minimize the floating body's offset. However, only the safety factor of mooring lines was considered. Harmony Search was utilized to optimize the mooring cost of FPSO in reference [11]. In this research, only three constraints were applied, including maximum platform offset, factor of safety for intact case top tension, and zero-degree angle of anchor. Besides GA, PSO and improved PSO were used to optimize the platform offset [12-15]. While factors such as floating body offset, mooring line tension, and seabed obstacles were taken into account, the role of sinker or clump weights was notably omitted from the analysis.

This study proposed an alternative computational optimization-based approach to investigate the behaviour of the mooring line considering the role of clump weights. In this effective method, the governing equations describing the mooring line profile and the continuity

and equilibrium conditions for cases involving clump weights are incorporated into the objective function and constraints of an optimization algorithm. Consequently, this approach provides an efficient and reliable computational tool for determining the mooring line configuration, making it highly suitable for the preliminary design stages of floating structures.

2. METHODS

The theory of mooring line analysis has been evolving since the early 20th century [4]. In the case of a single mooring line with the length l subjected to total transverse force H transferred from the floating body, the basic catenary equations define its equilibrium shape as follows:

$$z(l) = \frac{wl^2}{2EA_o} + \frac{Hl\tan\theta_0}{2EA_o} + a\sqrt{\left[1 + \left(\frac{l}{a} + \tan\theta_0\right)^2\right]} - \frac{a}{\cos\theta_0} \tag{1}$$

$$x(l) = \frac{Hl}{EA_0} + a \cdot arcsinh\left[\frac{l}{a} + tan(\theta_0)\right] - a \cdot arcsinh[tan(\theta_0)] \tag{2}$$

$$a = \frac{H}{w} \tag{3}$$

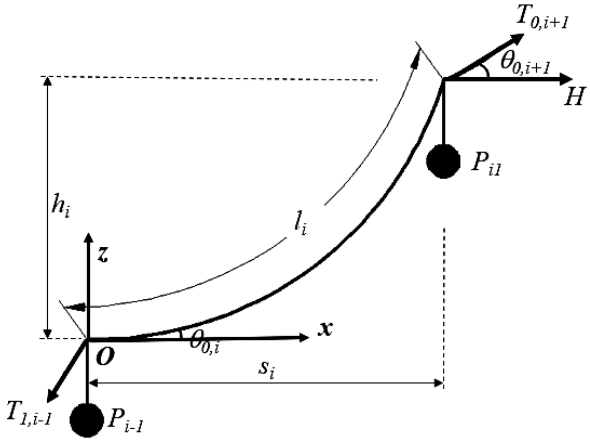


Figure 1. Diagram of a single mooring line segment.

Where x, z denote the spatial coordinates of the mooring line, θ_0 is the mooring line angle at the seabed. w is the unit weight of the mooring line in water. While E, A_0 are Young modulus of the material and the effective cross-section area of the mooring line. In addition, the length of a segment and the tension along the mooring line at the are calculated as shown in the following equations:

$$l(x) = a \cdot \left[\sinh\left(\frac{x}{a} + \operatorname{arcsin}(\tan(\theta_0)\right) - \tan(\theta_0)\right] \tag{4}$$

$$T(l) = H \cdot \sqrt{1 + \left(\frac{l}{a} + \tan(\theta_0)\right)^2} \tag{5}$$

In the case of a single mooring line with clump weights, the line is discontinuous at the attached points of each clump weight. Thus, each segment between two attached points is considered separately. Figure 1 shows the diagram of the i^{th} segment. The continuity of displacement and force equilibrium at the connection point must be satisfied. In this case, the mooring line is discretized, allowing each segment to be treated as an isolated single mooring line. Consequently, the lower endpoint of the segment effectively functions as a 'sinker,' acting as the fixed reference for the local profile. Conversely, the upper endpoint behaves as a 'floating body' responsible for sustaining the tensile loads and boundary conditions transferred from the overlying segment. Additionally, the geometric continuity and force balance at each attached point must be assured. The conditions at the attached point of the i^{th} clump weight are expressed as follows:

$$x_{1,i} = x_{0,i+1} \tag{6}$$

$$z_{1,i} = z_{0,i+1} \tag{7}$$

$$T_{1,i} \cdot cos\,\theta_{1,i} - T_{0,i+1} \cdot cos\theta_{0,i+1} = 0 \tag{8}$$

$$P_i + T_{1,i} \cdot sin\,\theta_{1,i} - T_{0,i+1} \cdot sin\theta_{0,i+1} = 0 \tag{9}$$

Where $x_{1,i}$, $z_{1,i}$, $T_{1,i}$ and $\theta_{1,i}$ denote the coordinates, line tension, and line incline angle at the upper end point of the segment i. While $x_{0,i+1}$, $z_{0,i+1}$, $T_{0,i+1}$ and $\theta_{0,i+1}$ denote the coordinates, line tension, and line incline angle at the lower end point of the segment $i+1$. P_i is the weight of the clump attached to the upper end point of the segment i.

Since, in the preliminary design, the water depth h, total transverse force H are the input data, while the total length l, the unit weight of the mooring line in water of the mooring line w, the number, and the positions of the clump weights are predetermined. Thus, the line inclination angles at the lower end point of each segment are the unknowns of the problem.

Once these inclination angles are determined, other parameters such as segment length, offset, and line tension can be deduced. In addition to the force balance conditions in Equation (9), the displacement continuity is constrained by the following summation equations:

$$\sum_{i=1}^{n} z_i(l_i) = h \tag{10}$$

Here $z_i(l_i)$ gives the vertical distance between the upper and lower ends of the segment.

As traditional numerical solvers, such as the Newton-Raphson method, often encounter convergence issues when dealing with the high non-linearity and sensitivity of multi-segment mooring systems, solving equations (6) through (10) simultaneously poses a significant computational challenge for conventional methods due to the high non-linearity and sensitivity of the system. To address this, the problem is reformulated as an optimization task. Instead of solving the complex system of algebraic equations directly, the problem is treated as finding the optimal set of decision variables that minimizes the closure error at the boundaries. By employing a numerical optimization algorithm, the solver iteratively adjusts the unknown parameters to converge towards a physically valid equilibrium state.

Consequently, this paper proposed an optimization-based method to solve the multi-segment mooring line problem. It should be noted that $\theta_{1,i}$ can be deduced from $\theta_{0,i}$ following equation (11).

$$\theta_{1,i} = \arctan(\tan(\theta_{0,i}) + \frac{w \cdot l}{H}) \tag{11}$$

Thus, $\theta_{0,i}$ is considered as the variable. The objective function is proposed based on the main idea that the continuity and equilibrium conditions are satisfied simultaneously, as shown in equation (12).

$$of = \left|P_i + T_{1,i} \cdot sin\,\theta_{1,i} - T_{0,i+1} \cdot sin\theta_{0,i+1}\right| + \left|\sum_{i=1}^{n} z_i(l_i) - h\right| \tag{12}$$

Constraints are proposed as follows:

- Threshold of the error of the force equilibrium at each point where a clump weight is attached:

$$P_i + T_{1,i} \cdot sin\,\theta_{1,i} - T_{0,i+1} \cdot sin\theta_{0,i+1} \leq 0.001 \tag{13}$$

- Threshold of the error of the water depth calculated by the sum of the vertical height of each segment:

$$\left|\sum_{i=1}^{n} z_i(l_i) - h\right| \leq 0.001 \tag{14}$$

- Geometric sag requirement to ensure that the mooring line segment is not simply a straight line, as follows:

$$s_i^2 + h_i^2 < l_i^2 \tag{15}$$

where, s_i, h_i, l_i are the offset, the height, and the length of the segment i, respectively, as shown in Figure 1. It should be noted that equation (12) represents a penalty function where both force and displacement residuals are minimized simultaneously. Although they have different physical units, the optimization goal is to drive both terms to zero, ensuring satisfaction of all physical constraints. Moreover, constraints in equations (13) and (14) are to ensure the accuracy of the optimal solution since the threshold of the error of the force equilibrium and the water depth are set to be strictly approximately 0 (less than 0.001). In other words, the two constraints above force the objective function $0f$ to approach zero.

Finally, using the optimization algorithm, the incline angles of each segment can be found such that the objective function $0f$ shown in equation (12) approaches zero ($of \to 0$). Consequently, the force balance and displacement continuity are satisfied. And thus, a complete static configuration of the multi-segment mooring system is successfully determined.

3. EXAMPLES

3.1. Example 1: single mooring line with one clump weight

In this example, a single mooring line with one clump weight is considered. The floating object in a water depth $h = 9.5m$ subjects to a constant transverse force of $H = 20\ kN$. One clump weight is attached. The distance from the sinker to the clump weight is $5m$. Details of the clump weight and the mooring line are shown in Table 1.

Table 1. Input data for Example 1.

Parameter	Value
Mooring line diameter, $d\ (m)$	0.1
Mooring line length, l(m)	20.0
Steel Modulus, E(kN/m^2)	2.1e8
Clump weight, P(kN)	5

While recent metaheuristic algorithms may offer advantages in highly complex scenarios, this study employs Particle Swarm Optimization (PSO) and Genetic Algorithm (GA). These traditional algorithms have been widely used in various engineering problems and are proven to have high reliability. Given the relatively low-dimensional optimization space of the 2D mooring line configuration, PSO and GA are highly efficient and sufficient to provide accurate solutions without complex parameter tuning.

In order to solve this problem, the variables are limited to the boundary from 0 to $\pi/3$ radians. Both algorithms are set up with the same parameters:

- Number of particles: $n_p = 200$;

- Number of iterations: $n_{ite} = 300$;

- Number of runs: $n_{run} = 50$.

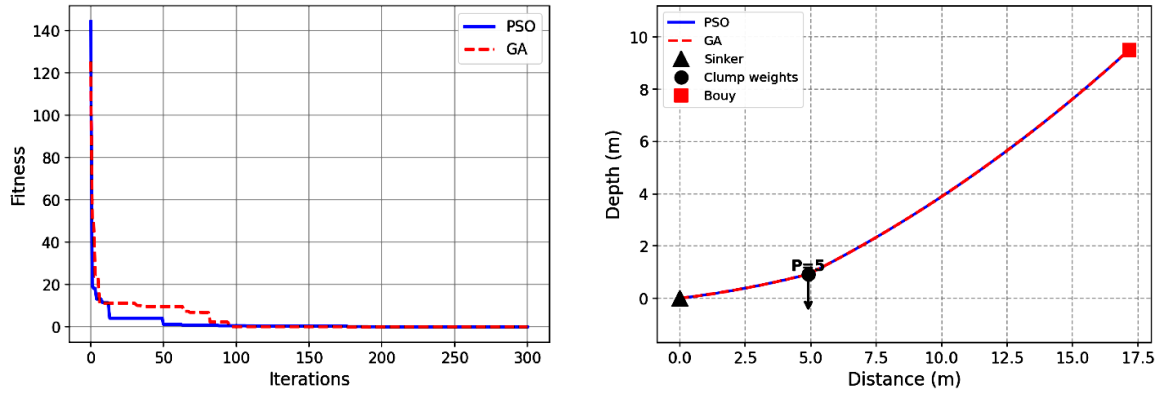
For algorithm-specific parameters, different values were used according to the characteristics of each method as shown in Table 2. It should be noted that these parameters are the standard ones of each algorithm.

Table 2. Specific parameters of PSO and GA.

Algorithm	Parameter	Value/Setting
PSO	c_1	2.0
	c_2	2.0
	w	Linearly decreasing ($0.9\ to\ 0.4$)
GA	P_c	0.9
	P_m	0.05
	σ	0.02

The results are shown in Table 3. It can be seen that two algorithms give the same results (maximum error of 0.03%. Also, the values of the objective functions are 2^{-9} and 2^{-4} respectively shows that the force balance and the continuity conditions are satisfied. Therefore, it can be concluded that the standard recommended parameters of PSO and GA listed above are highly suitable for this approach, rendering a dedicated sensitivity analysis for parameter tuning unnecessary. Besides, Figure 2a shows that PSO converges faster than GA (around the iteration of 50 vs 100).

In addition, the results clearly show that once the incline angles at the lower end of each segment are found, the other parameters of the mooring line are obtained. Figure 2b depicts the shape of the mooring line. Discontinuity is observed at the point where the clump weight is attached. The mooring line is divided into two separate segments.



(a) (b)

Figure 2. Comparison of PSO and GA in the case of one clump weight (a) Convergence curve, (b) Mooring line shape.

Table 4 shows the comparison between the results obtained from the optimization-based method and FEA using Sap2000 in reference [9]. In this model, the mooring line is discretized using cable elements with clump weights applied as concentrated vertical point loads. Large geometric deformations are accurately captured using the Nonlinear solver combined with the P-Delta Plus option. Results obtained from this FEA model include mooring line tension, uplift and horizontal force of the sinker and the deformed shape of the mooring line. It is highlighted that the difference between the two methods is negligible. Therefore, the optimization-based method is validated.

Table 3. Results of example 1.

No	Parameters	Value		
		PSO	GA	Error (%)
1	$\boldsymbol{\theta_{0,1}}$**(rad)**	0.120	0.120	0.0
2	$T_{0,1}$(kN)	20.14	20.14	0.0
3	$\theta_{1,1}$(rad)	0.250	0.250	0.0
4	$T_{1,1}$(kN)	20.64	20.64	0.0
5	$\boldsymbol{\theta_{0,2}}$**(rad)**	0.468	0.468	0.0
6	$T_{0,2}$(kN)	22.40	22.41	0.0
7	$\theta_{1,2}$(rad)	0.737	0.737	0.0
8	$T_{1,2}$(kN)	27.01	27.02	0.03
9	S(m)	17.16	17.16	0.0
10	Of	3e-9	2e-4	

Table 4. Comparison of the Optimization-based method and FEA.

No	Parameters	Value		
		Optimization based	FEA	Error (%)
1	$\theta_{0,1}$(rad)	0.120	0.120	0.0
2	$T_{0,1}$(kN)	20.17	20.10	0.3
3	$T_{1,2}$(kN)	27.01	27.00	0.04
4	S(m)	17.16	17.16	0.00

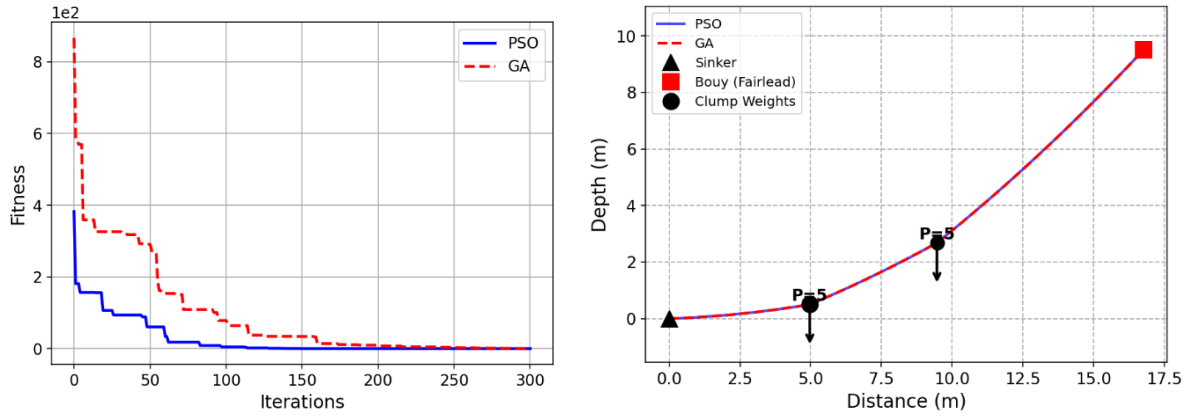
3.2. Example 2: single mooring line with two clump weights

In this example, a single mooring line with two clump weights is considered. The second clump weight is placed at a distance of $5m$ from the first one, measured along the mooring line. The transverse load and the other parameters are the same as in example 1.

Figure 3b illustrates the comparison of the mooring line shape obtained from PSO and GA. The parameters of both algorithms are the same as in section 3.1. As in example 1, the results obtained using the two algorithms in this example show a noticeable good agreement. Additionally, PSO still has a higher convergence speed than GA. The difference between the results from the optimization-based method and FEA is also negligible as shown in Table 5. Therefore, it can be concluded that an optimization-based method is a reliable solution to calculate a single mooring line with clump weights.

Table 5. Comparison of the Optimization-based method and FEA.

No	Parameters	Value		
		Optimization based	FEA	Error (%)
1	$\theta_{0,1}$(rad)	0.0336	0.0335	0.3
2	$T_{0,1}$(kN)	20.01	20.01	0.0
3	$T_{3,2}$(kN)	29.31	29.31	0.0
5	S(m)	16.77	16.77	0.0



(a) (b)

Figure 3. Comparison of PSO and GA in the case of two clump weights (a) Convergence curve, (b) Mooring line.

4. CONCLUSIONS

This study proposed an optimization-based approach to solve a multi-segment problem of a single mooring line, considering clump weights attached to the line. The primary conclusions drawn from this research are as follows:

- The conversion of the non-linear algebraic system into a single-variable optimization task (seabed inclination angle) significantly enhances computational efficiency and solution accuracy.

- Force equilibrium and geometric continuity at the attachment points of clump weights are strictly satisfied by minimizing the objective function toward zero.

- The PSO algorithm demonstrates a clear advantage over GA in terms of convergence rates and reliability, especially for sensitive multi-segment problems.

- The strong agreement between the results of the proposed optimization method and conventional FEA confirms the reliability of the approach for preliminary design stages of floating structures.

- Incorporating specific geometric constraints and precise elastic elongation calculation ensures that the numerical model accurately reflects the physical behaviour of mooring lines under high-tension conditions.

This approach provides an efficient and reliable computational tool for determining mooring line configuration, making it highly suitable for the preliminary design stages of floating structures. For 3D multi-line systems, this 2D approach can be applied independently to each line. By vectorially superimposing these individual restoring tensions, global station-keeping performance can be quickly determined, completely bypassing the computational bottlenecks of coupled 3D FEA. Future research will focus on 3D and complex problems of the mooring system.

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