



RELIABILITY-BASED MULTI-OBJECTIVE OPTIMIZATION OF LAMINATED PLATES USING GDE3 AND ISOGEOMETRIC ANALYSIS

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Abstract. Laminated composite plates are widely used in lightweight structures owing to their high stiffness-to-weight ratio and flexible design capability. However, uncertainties in loads, material properties, and design parameters may significantly affect their mechanical performance and are often not fully addressed in deterministic optimization. This study presents a reliability-based multi-objective optimization framework for the bending design of laminated composite plates under uncertainty. The structural response is evaluated using Isogeometric Analysis combined with the First-order Shear Deformation Theory, allowing geometry and transverse shear deformation to be represented in a unified numerical model. Ply thicknesses are design variables, while maximum transverse deflection and structural mass are minimized as conflicting objectives. The First-order Reliability Method is used to evaluate the reliability index, and the Generalized Differential Evolution 3 algorithm is used to obtain Pareto-optimal solutions. Numerical results show that uncertainties affect the feasible design region and the trade-off between mass, deflection, and reliability. The framework supports reliability-informed design of lightweight composite plate structures.

Keywords: Multi-objective optimization, Laminated composite plate, Isogeometric Analysis FSDT, GDE3 algorithm, Reliability index FORM, Ply thickness.

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1. INTRODUCTION

In the context of modern engineering material development, multi-layer composite plates have gained significant attention for use in structures that demand both light weight and high durability, particularly in aerospace, automotive, and marine applications [1]. However, due to inherent uncertainties-such as manufacturing tolerances, material variability, and complex loading conditions-conventional design approaches often fall short in ensuring effectiveness and reliability in real-world applications. Furthermore, the intrinsic complexity of multi-layer configurations and fiber orientations poses significant challenges to the design optimization process, especially when addressing conflicting objectives [2], such as minimizing structural weight while limiting deflection.

In recent years, Isogeometric Analysis (IGA) has emerged as a powerful tool in numerical simulation, particularly effective for structural problems with complex geometries or high accuracy requirements [3]. In contrast to traditional Finite Element Methods (FEM), IGA utilizes the same basis function- typically NURBS- for both the geometric model and the displacement field, thereby eliminating geometric errors and improving convergence accuracy [4]. When integrated into the optimization process, IGA enables smooth simulation of the bending behavior of laminate composite plates while ensuring geometric consistency throughout the design update process [5]. Moreover, when combined with reliability analysis, IGA serves as a foundational tool for developing precise limit state functions, thereby enhancing the reliability of evaluations and improving the value of multi-objective optimization in uncertain environments [6]. Consequently, the integration of IGA not only enhances simulation efficiency but also plays a pivotal role in developing reliable optimization models for multi-layer composite structures.

The GDE3 algorithm is a multi-objective evolutionary solution developed to address multi-objective problems-specifically in contexts where classical algorithms like NSGA-II or SPEA2 begin to show limitations in performance [7]. Whereas the non-dominance ranking approach of NSGA-II, GDE3 extends the Differential Evolution (DE) algorithm for multi-objective optimization, integrating a Pareto dominance-based selection mechanism with an improved "crowding distance" measure as a central selection criterion, thus directly improving the distribution of solutions along the Pareto front. This Pareto dominance-based approach is also extended to handle constraints, ensuring that feasible individuals are prioritized over infeasible ones-allowing GDE3 to efficiently solve problems with complex constraints. GDE3 has been applied across many engineering fields requiring multi-objective optimization. In the field of structural design, it has proven particularly effective when applied to truss optimization problems, achieving simultaneous reductions in weight and improvements in dynamic performance [8]. In electrical systems, an improved version of GDE3 has been proposed to solve the Optimal Reactive Power Dispatch (ORPD) problem with multiple objectives, such as reducing power losses and improving voltage stability [9]. Although GDE3 has demonstrated superior performance in numerous multi-objective optimization problems-particularly in structural geometric design and engineering systems with a large number of objectives-the application of GDE3 combined with reliability analysis in the design of plate structures remains limited. Most current studies focus primarily on pure geometric optimization or fail to account for uncertainties in design variables and operating conditions. The lack of integrated studies between GDE3 and reliability assessment methods (such as FORM) in uncertain environments creates a significant gap that needs to be addressed in order to improve both safety and efficiency in the real-world design of composite plates.

Building upon the identified limitations, this study proposes a multi-objective optimization framework integrated with reliability analysis for laminate composite plate structures. The GDE3 algorithm is adopted to identify Pareto-optimal design solutions addressing two conflicting objectives: minimizing deflection and reducing weight. Mechanical analysis is conducted using Isogeometric Analysis (IGA) in conjunction with First-Order Shear Deformation Theory (FSDT), enabling accurate simulation of bending behavior in multi-layered plates. Design reliability is quantified using the β -index based on the First-Order Reliability Method (FORM), incorporating uncertainties related to loading conditions, material properties, and design variables. The results offer not only a quantitative assessment of uncertainty effects on structural performance but also contribute to the development of a robust design database for engineering applications in uncertainty-prone environments.

2. EXPANSION EQUATIONS AND THEORY

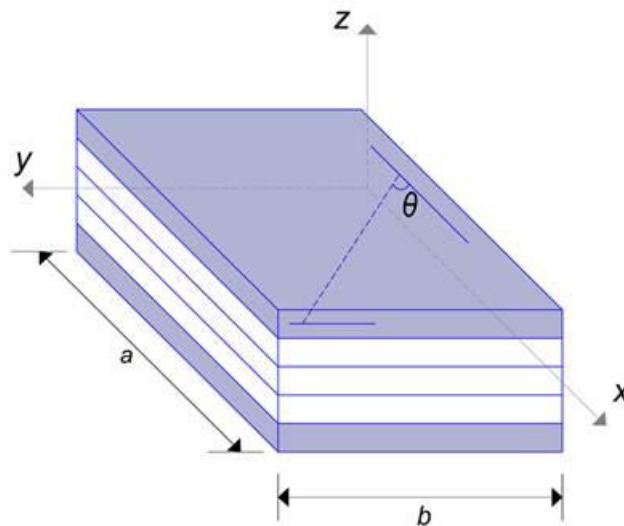


Figure 1. Schematic diagram of the laminated plate structure.

Figure 1 illustrates the structure of a multilayer composite laminate in the form of a rectangular plate with in-plane dimensions $a \times b$ and a total thickness h . The laminate comprises multiple plies stacked along the thickness direction z , with each ply potentially having a distinct fiber orientation. The angle θ denotes the fiber orientation angle of each ply relative to the x -axis, allowing for the modelling of composite structures with symmetric, antisymmetric, or arbitrary configurations.

The three-dimensional Cartesian (x, y, z) coordinate system is employed to describe the geometry and mechanical analysis of the plate. The thickness of each layer can be uniform or vary depending on the design variable, in accordance with the optimization structure. In this study, the composite plate is assumed to have standard boundary conditions and is subjected to a vertically distributed load (along the z -axis), allowing for the evaluation of the bending performance of the structure in practical configurations commonly encountered, such as technical floors and coatings.

2.1. First-Order Shear Deformation Theory (FSDT)

First-Order Shear Deformation Theory (FSDT) allows for a more accurate modeling of the behavior of thick plate structures compared to classical thin plate theory. In FSDT, the displacement at any point within the plate is expressed as follows:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z\phi_x(x, y), \\ v(x, y, z) &= v_0(x, y) + z\phi_y(x, y), \\ w(x, y, z) &= w_0(x, y), \end{aligned} \tag{1}$$

Where, u_0, v_0, w_0 represents the displacements at the neutral surface, and ϕ_x, ϕ_y are the rotations of the cross-section perpendicular to the x and y axes. The plate deformation consists of two main components: membrane bending deformation and shear deformation. Shear deformation is linear with respect to the thickness and non-vanishing, addressing the limitations of classical models in the case of thick plates.

The material model of the multilayer laminate plate is constructed based on the generalized Hooke's law, through the thickness-integrated stiffness matrix, which includes the matrices A, B, D and A_s as follows:

$$\sigma = \begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & A_s \end{bmatrix} \begin{Bmatrix} \epsilon_0 \\ \kappa \\ \gamma_s \end{Bmatrix} \tag{2}$$

Where N, M, Q is the membrane force, moment, and shear force; ϵ_0, κ and γ_s are the membrane strain, curvature, and average shear strain, respectively [11].

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{N_l} (\bar{Q}_{ij})^{(k)} (z_k - z_{k-1}), \quad i, j = 1, 2, 3 \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{N_l} (\bar{Q}_{ij})^{(k)} (z_k^2 - z_{k-1}^2), \quad i, j = 1, 2, 3 \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{N_l} (\bar{Q}_{ij})^{(k)} (z_k^3 - z_{k-1}^3), \quad i, j = 1, 2, 3 \\ (A_s)_{\alpha\beta} &= \sum_{k=1}^{N_l} k_s^{(k)} (\bar{Q}_{\alpha\beta})^{(k)} (z_k - z_{k-1}), \quad \alpha, \beta = 4, 5 \end{aligned} \tag{3}$$

2.2. Isogeometric Analysis (IGA)

Isogeometric Analysis (IGA) is a modern numerical method that tightly integrates CAD modelling with analysis modelling. Compared with the Finite Element Method (FEM), IGA uses the same basis functions, NURBS (Non-Uniform Rational B-Splines) or B-splines, to describe both the geometry and the displacement field. As a result, IGA eliminates geometric errors and achieves higher accuracy in simulations. The shape of the plate surface is defined by a degree- p NURBS interpolation function, which is expressed by the following equation:

$$\mathbf{x}(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m R_{i,j}^p(\xi, \eta) \mathbf{P}_{i,j} \quad (4)$$

Where $R_{i,j}^p$ is the NURBS basis function; $\mathbf{P}_{i,j}$ are the control points; and ξ, η is the parameter in the NURBS reference system.

The displacement field $\mathbf{u}(\xi, \eta)$ is also approximated using the same basis functions:

$$\mathbf{u}(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m R_{i,j}^p(\xi, \eta) \mathbf{d}_{i,j} \quad (5)$$

Where $\mathbf{d}_{i,j}$ is the degree-of-freedom vector at the control point.

The stiffness matrix and load vector are constructed in a manner similar to the Finite Element Method (FEM), but are derived through numerical integration over the NURBS parameter space.

2.3. Multi-objective Optimization with GDE3 and Reliability Assessment

The multi-objective optimization problem (Multi-objective Optimization – MOO) is generally formulated as follows:

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{X}} \quad & \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})] \text{ constraints: } g_j(\mathbf{x}) \leq 0, \quad j = 1, 2, \dots, J \\ & h_k(\mathbf{x}) = 0, \quad k = 1, 2, \dots, K \end{aligned} \quad (6)$$

Where $\mathbf{x} \in \mathbb{R}^n$ is the vector of design variables; \mathcal{X} is the feasible design space (including upper/lower bounds and geometric constraints); $\mathbf{F}(\mathbf{x})$ is the set of objective functions to be optimized simultaneously; and $g_j(\mathbf{x}), h_k(\mathbf{x})$ are the inequality and equality constraints. In this problem, the design vector is specified as follows:

$$\mathbf{x} = [t_1, t_2, \dots, t_N, \theta_1, \theta_2, \dots, \theta_N]^T \in \mathbb{R}^{2N} \quad (7)$$

Where t_i is the thickness of the i ply; θ_i is the fiber orientation angle of the i ply; and N is the total number of plies in the laminate.

The two objective functions typically considered simultaneously are:

$f_1(\mathbf{x})$ is the structural mass (or volume), which needs to be minimized;

$f_2(\mathbf{x})$ is the maximum deflection at the center of the plate, which also needs to be minimized.

The quantitative expression can be written as:

$$f_1(\mathbf{x}) = \rho \sum_{i=1}^N t_i \cdot A, (\text{total mass, where } A \text{ is the area of the plate}) \quad (8)$$

$$f_2(\mathbf{x}) = w_{max}(\mathbf{x}) = \max_{(x,y)} w(x, y; \mathbf{x}), (\text{the maximum deflection depends on } (t_i, \theta_i)) \quad (9)$$

The constraints may encompass:

Technical constraints on thickness $t_i^{min} \leq t_i \leq t_i^{max}$;

Ply angle within a reasonable range $\theta_i \in [-90^\circ, +90^\circ]$;

The obtained solution set after optimization is a Pareto set, where no solution exists that is simultaneously better in all objectives.

GDE3 (Generalized Differential Evolution 3) is a multi-objective evolutionary algorithm extended from Differential Evolution (DE) to improve convergence and the distribution of solutions along the Pareto front in multi-objective problems. In contrast to algorithms such as NSGA-II, which rely solely on non-dominance ranking, GDE3 combines a Pareto dominance-based selection mechanism with crowding distance to assess the quality of the solution set. Specifically, after generating offspring using DE operators, GDE3 merges the parent and offspring populations, filters them based on Pareto dominance, and then uses crowding distance to maintain diversity among non-dominated solutions.

With this mechanism, GDE3 avoids the complex computation of hypervolume while still effectively distinguishing individuals near the Pareto front, even in many-objective problems. In this study, GDE3 is applied to a bi-objective optimization problem involving the minimization of maximum deflection and structural weight of a laminated composite plate. The resulting Pareto-optimal set offers designers multiple flexible solutions, enabling a trade-off between mechanical performance and structural reliability under uncertainty.

Once the optimization has been performed, the obtained Pareto solution set represents design alternatives that are efficient in both mechanical objectives: minimizing weight and limiting deflection. However, in an uncertain working environment, each alternative must be assessed for reliability using the FORM (First-Order Reliability Method) [10].

The present work formulates the limit state function as follows:

$$g(\mathbf{X}) = w_{allow} - w(\mathbf{X}) \quad (10)$$

In which, $\mathbf{X} = \{E, F\}$ is the set of random variables including the modulus of elasticity and load; $w(\mathbf{X})$ is the maximum deflection at the test point (e.g., the center of the plate), calculated through the IGA-FSDT analysis; w_{allow} is the allowable deflection according to the design standards.

The value of the function $g(\mathbf{X}) > 0$ represents the safety state, while $g(\mathbf{X}) \leq 0$ corresponds to the risk of exceeding the allowable deflection. The transformation from the physical space to the standard space is carried out as follows:

$$Z_i = \frac{X_i - \mu_i}{\sigma_i}, \quad i = 1, 2, \dots, n, \quad (11)$$

and the reliability index β_r is determined by finding the closest point to the origin in the standard space such that $g(\mathbf{X}(Z)) = 0$. From this, the approximate failure probability is calculated as follows:

$$P_f \approx \Phi(-\beta_r) \tag{12}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Thus, the overall process consists of two stages: GDE3 is employed to generate Pareto-optimal design alternatives based on performance objectives, while FORM is used to conduct post-validation of the reliability of each alternative. Solutions that meet the desired reliability threshold can be further filtered to establish a refined design set, supporting decision-making under realistic uncertain conditions.

3. EXAMPLE AND DISCUSSION

3.1. Validation of the numerical results

In order to verify the accuracy of the proposed finite element model and the solution procedure based on IGA–FSDT, a standard static bending problem of a $[0^\circ/90^\circ/0^\circ]$ laminated composite plate subjected to a sinusoidal distributed load is employed. The obtained results are then compared with those reported in the literature. The geometry of the problem is a rectangular plate with dimensions $a \times b$, subjected to a distributed load perpendicular to the plate surface, with the following form: $q(x, y) = q_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$. This is a classic form of distributed load commonly used in validation studies of composite plates [11], ensuring static equilibrium conditions and accurately reflecting the bending response of the structure in the case of simplified load profile boundaries. In this example, the material is taken as the isotropic material properties of the laminated plate, with the following parameters: $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$, $\rho = 1$. The results are evaluated through the non-dimensional central deflection of the plate, defined by the following formula:

$$\bar{w} = \frac{E_2 h^3}{a^4 q_0} w\left(\frac{a}{2}, \frac{b}{2}, 0\right) \tag{13}$$

Table 1. Comparison of the non-dimensional central deflection of the laminated composite plate.

a/h	Reference study	$\bar{w} \times 10^2$	Relative error (%)
4	Reddy [11]	1.776	–
	This study	1.786	0.55
10	Reddy [11]	0.669	–
	This study	0.673	0.62
20	Reddy [11]	0.491	–
	This study	0.495	0.80
100	Reddy [11]	0.434	–
	This study	0.436	0.50

The results presented in Table 1 show a very good agreement between the current numerical model and the reference data from Reddy [11] in predicting the non-dimensional central deflection of a laminated composite plate subjected to a sinusoidal distributed load. For different geometric ratios $a/h = 4, 10, 20, 100$, the relative error between the current model and

the reference data remains low, ranging from 0.50% to 0.80%, demonstrating that the finite element model is capable of accurately simulating different thickness configurations in the plate.

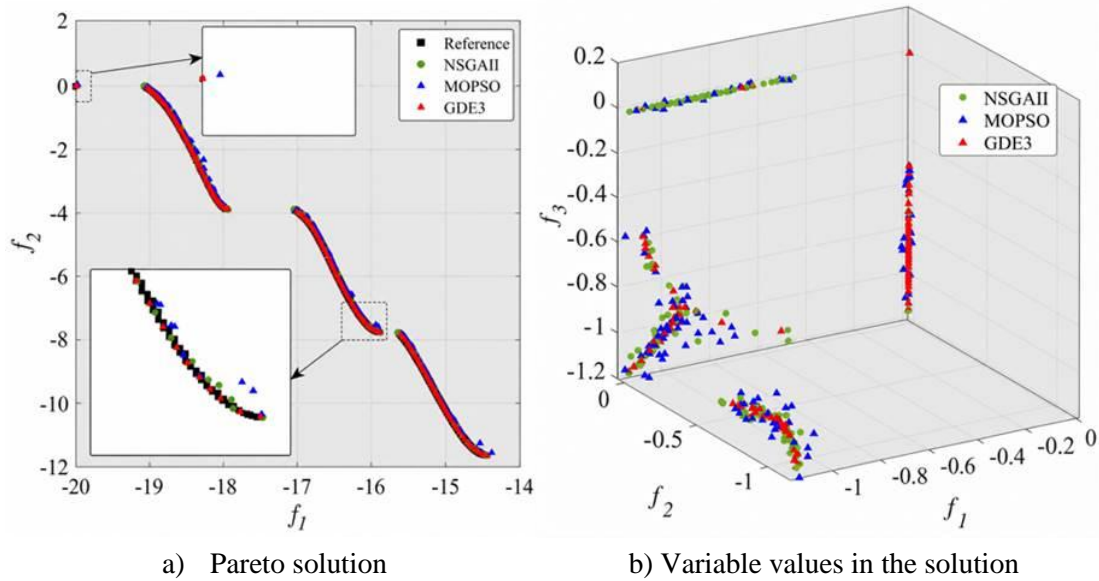


Figure 2. Comparison of the Pareto front for the Kursawe problem.

Finally, to demonstrate the accuracy and reliability of the Pareto front in this study, we performed a comparative analysis using the famous Kursawe problem – a benchmark problem commonly used to evaluate the performance of multi-objective optimization algorithms in a nonlinear, non-convex space with multiple broken regions. Figure 2 presents the comparison results between three algorithms: NSGA-II [12], MOPSO [13], and GDE3. The characteristic parameters of the algorithm are set to their recommended default values, as shown in Table 2. In Figure 2(a), the distribution of the solutions in the objective space (f_1, f_2) is shown, while Figure 2(b) presents the corresponding design space (x_1, x_2, x_3). In Figure 2, all algorithms approach the reference Pareto front; however, GDE3 (red triangle) shows superior coverage and accuracy in approximation, especially in the kinked segments and strongly curved regions of the Pareto front (as zoomed in on the two inserted areas). MOPSO (green triangle) tends to distribute solutions sparsely, while NSGA-II (green star) closely follows the boundary but lacks the evenness and stability of GDE3. In Figure 2(b), the differences in solution quality are more clearly evident in the design space. Solutions from GDE3 are tightly and controllably distributed along the boundary regions and design surface, indicating that this algorithm maintains better diversity across the entire search space. MOPSO and NSGA-II algorithms appear to result in solution clusters that are either skewed or lack uniform distribution. It can be seen that GDE3 is not only effective in generating an accurate Pareto front, but also ensures even coverage and distribution in both the design and objective spaces. This is a key factor when applied to real-world optimization problems, where stability and the ability to find diverse solutions play a crucial role.

Table 2. Configuration Parameters of Multi-Objective Optimization Algorithms

Algorithm	Population Size	Maximum Function Evaluations	Parameters
NSGA-II	100	150000	SBX: $\eta_c = 20$, Mutation: $\eta_m = 20, p_c = 1, p_m = \frac{1}{n}$
MOPSO	100	150000	$\omega = 0.4, c_1 = c_2 = 1.0$
GDE3	100	150000	$CR = 0.9, F = 0.5$

3.2 Multi-Objective Optimization

In this optimization problem, we consider a laminate composite plate subjected to a sinusoidal distributed load, with material properties and ply configuration as outlined in Section 3.1. The two objective functions are minimizing weight and reducing the maximum deflection at the plate center, with no additional constraints considered. To investigate the effect of boundary conditions, the following boundary combinations are independently optimized: SSSS, CCCC, CFFF, CFCF, CCCF. Each case results in a different Pareto set, representing the trade-off between weight and deflection.

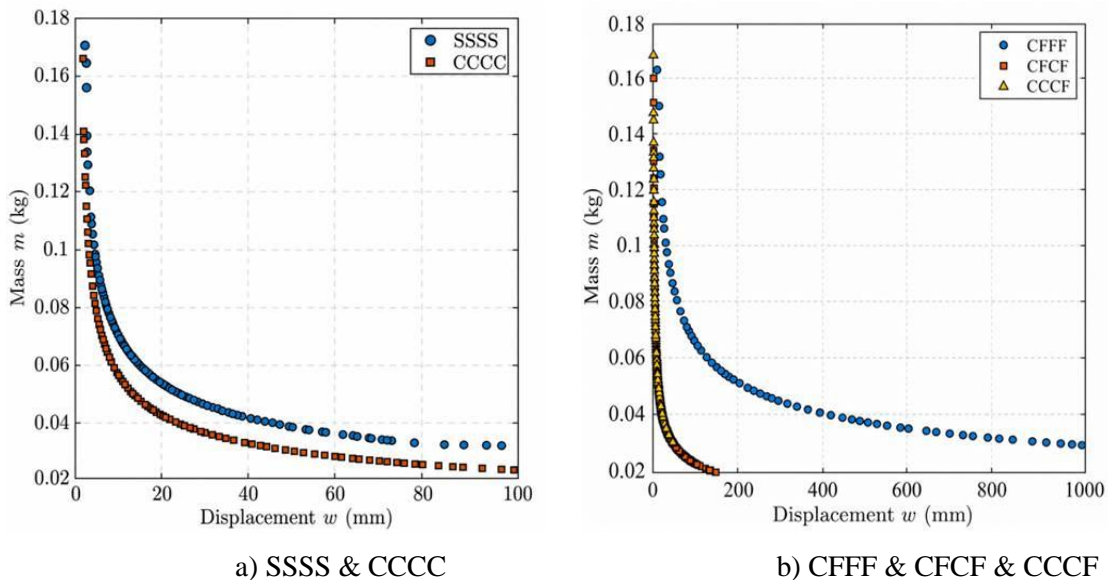


Figure 3. Pareto Optimal Solution Set for Different Boundary Condition Combinations: a) SSSS & CCCC; b) CFFF & CFCF & CCCF.

For Figure 3(a), when transitioning from the SSSS boundary condition to the CCCC boundary condition, the Pareto front shifts to the left – meaning that for the same deflection, the plate with the CCCC boundary condition requires less weight than the SSSS boundary condition. This is consistent with intuition: tighter boundary conditions increase bending resistance, which reduces weight. Figure 3(b) shows that the three combinations of CFFF, CFCF, and CCCF cause the Pareto front to shift further to the left as the number of fixed edges increases, reflecting an increase in stiffness. Overall, adjusting the boundary conditions allows

the designer to flexibly balance weight and deflection: more fixed edges \rightarrow less weight required to reduce deflection.

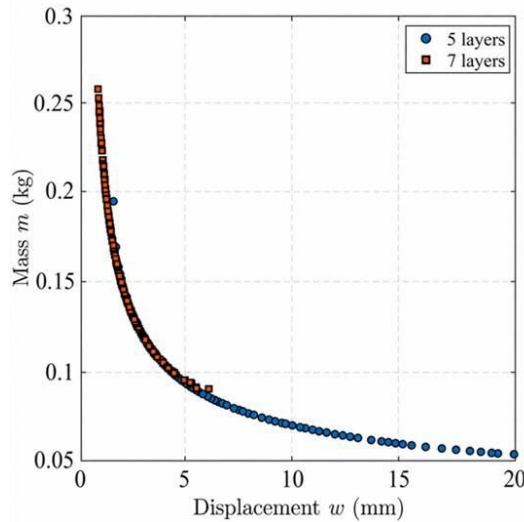


Figure 4. Comparison of the Pareto Front between the 5-ply and 7-ply configurations for the same multi-layered plate bending problem.

Subsequently, the multi-objective optimization results with an increased number of layers are investigated. Figure 4 shows the shift in the Pareto front when increasing the number of ply layers from 5 to 7 in the same weight-deflection optimization problem. Overall, the solution set for the 7-ply configuration (orange square) is positioned above and slightly to the left of the 5-ply configuration (green circle). This is an inevitable result of each ply having a thickness within the range of $t_{min} - t_{max}$; as the number of layers increases, the overall thickness increases, leading to an increase in minimum weight. Although the multi-layer configuration allows finer adjustments to fiber angles through layer resolution, the effect of reducing deflection per unit weight is not sufficient to offset the additional weight.

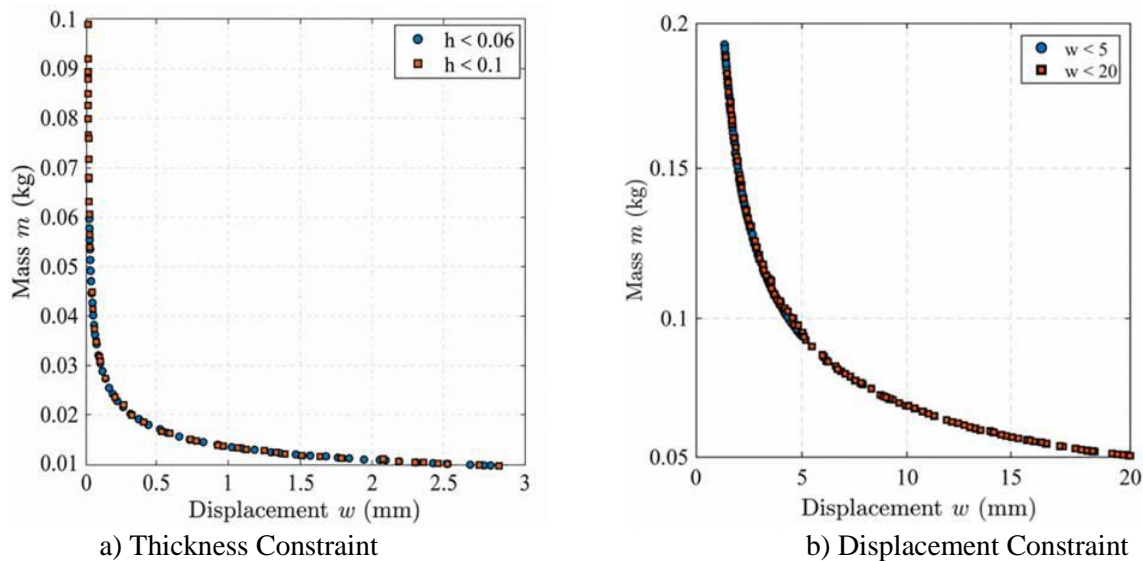


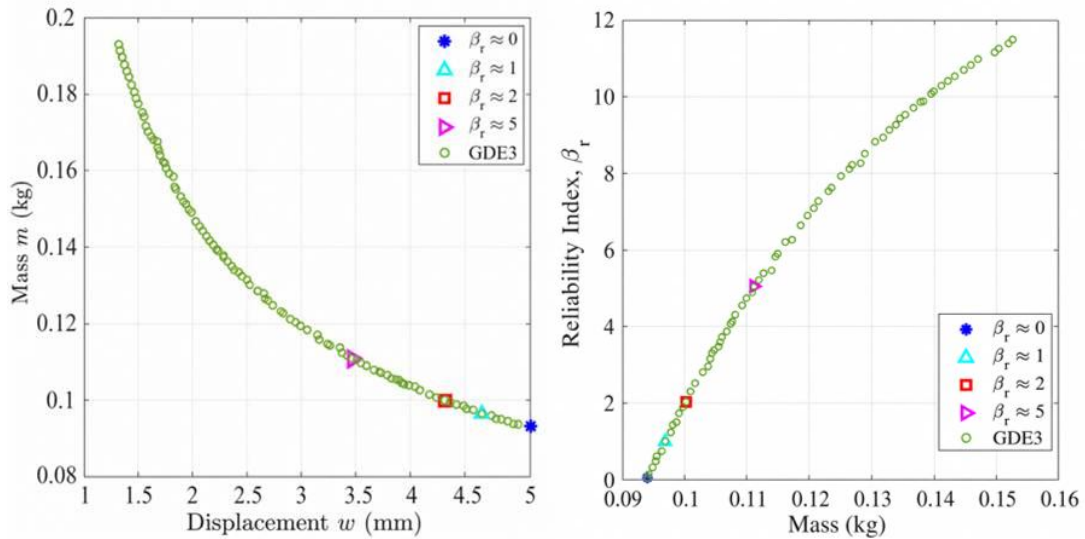
Figure 5. Pareto Front Considering Additional Constraints in the Same Multi-Layered Plate Bending Problem: a) Thickness Constraint; b) Displacement Constraint.

The subsequent analysis, the multi-objective optimization problem with additional constraints is considered. Figure 5(a) compares two Pareto solution sets obtained when the total thickness of the layers h is limited to two different values ($h < 0.06$ and $h < 0.1$). Both sets (green dots and orange squares) exhibit the basic trend of the Pareto boundary: with the goal of reducing weight, displacement increases rapidly in the small region and then stabilizes. However, when a larger total thickness is allowed, the orange solution set expands to higher weight values for the same displacement, reflecting that a wider thickness limit allows for additional material to reduce deflection, but at the cost of a higher optimal weight. In contrast, with a stricter limit, the model is forced to select a thinner plate, resulting in a smaller weight but larger displacement at the Pareto boundary's end. This result underscores the importance of thickness constraints in design. Figure 5(b) illustrates the Pareto front when applying a displacement constraint - specifically comparing two limits, $w < 5$ mm, and $w < 20$ mm. When the displacement limit is stricter ($w < 5$ mm), the solution set must shift upward and to the left on the graph, indicating that to stay within the 5 mm limit, the multi-layered plate must use more material (higher weight) compared to the case allowing up to 20 mm of deflection. This result shows that displacement constraints directly affect the trade-off between weight and deflection: the tighter the displacement limit, the higher the optimal weight, while relaxing the limit allows for a reduction in weight but at the expense of greater deflection. This is a key basis for guiding design, as the requirements for deformation limits need to be considered alongside the objective of material savings.

3.3 Multi-Objective Optimization and Reliability Assessment

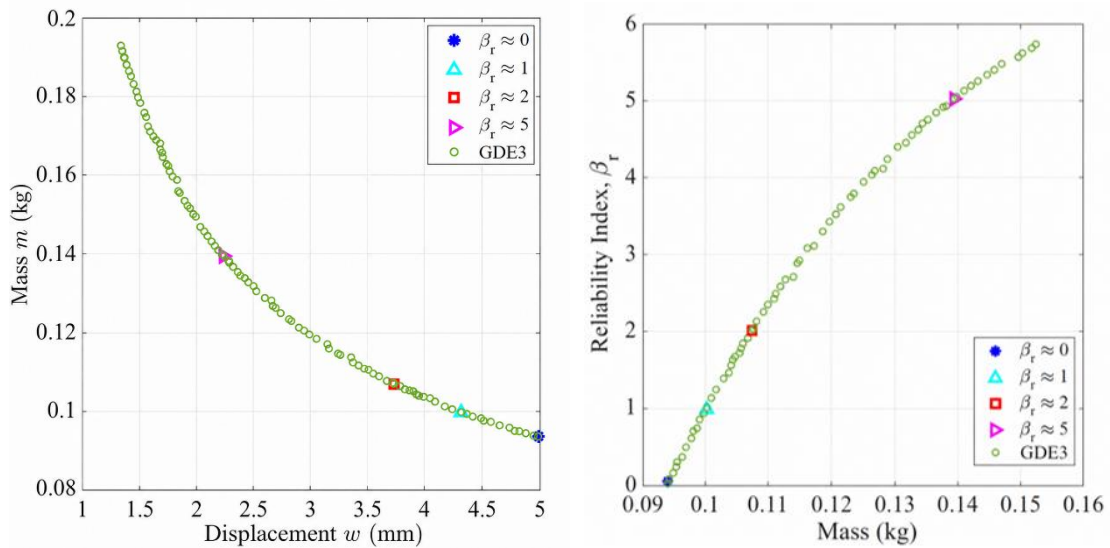
To ensure both performance and safety for multi-layer composite structures under uncertain conditions, this section outlines the integration of multi-objective optimization with reliability analysis. Initially, the GDE3 algorithm is employed to generate the Pareto solution set, balancing two objectives: weight reduction and deflection minimization. Subsequently, each Pareto solution is assessed for reliability β_r , using the FORM method, with the standard deviation of the load mean value and elasticity modulus treated as random variables. The final outcome is a "reliable" Pareto front, where each design alternative is not only mechanically optimal but also meets safety requirements under material and load uncertainties.

Figure 6(a) illustrates the Pareto front obtained by GDE3 (green dots) with reference points ($\beta_r \approx 0, 1, 2, 5$) evaluated under random variables with a standard deviation equal to 5% of the mean value. The results indicate that the algorithm not only identifies optimal trade-offs in terms of performance (weight and deflection) but also covers the desired levels of reliability. Specifically, the inclusion of reference points within the Pareto front confirms that GDE3 is capable of generating designs that precisely meet different reliability requirements. Figure 6(b) depicts the relationship between structural weight and the corresponding reliability index β_r of the Pareto-optimal solutions. The observed trend shows that the reliability index β_r increases with structural weight, suggesting that additional material contributes to enhanced structural reliability. The uniform distribution of green dots further confirms that GDE3 maintains a good balance between solution diversity and convergence toward the target reliability spectrum. As a result, designers are provided with a comprehensive set of options to select an appropriate trade-off between structural weight and safety in accordance with practical requirements.



a) Displacement constraints at reference nodes β_r , b) Reliability index corresponding to structural weight

Figure 6. Optimal results obtained by GDE3 under the deflection constraint $w < 5$ mm, (standard deviation equal to 5% of the mean value): (a) Pareto front of weight–deflection with reference points at designated levels β_r , (b) Relationship between structural weight and reliability β_r index for the entire set of feasible solutions.



a) Displacement constraints at reference nodes β_r , b) Reliability index corresponding to structural weight

Figure 7. Optimal results obtained by GDE3 under the deflection constraint $w < 5$ mm (standard deviation equal to 10% of the mean value): (a) Pareto front of weight–deflection with reference points at different levels β_r ; (b) relationship between structural weight and the reliability index β_r for the entire set of feasible solutions.

Figure 7 presents the optimization results obtained using GDE3 under the deflection constraint $w < 5$ mm, with the standard deviation of the random variables increased to 10% of the mean value (as compared to 5% in Figure 6). Notably, the reference points are now located closer to the upper region of the solution set Figure 6(a), reflecting a tendency for higher structural weight to be required in order to achieve the same reliability index β_r when the uncertainty σ increases. In Figure 7(b), the relationship between structural weight and the reliability index β_r , exhibits a steeper rising trend compared to Figure 6(b). This implies that as uncertainty increases (from 5% to 10%), a given structural weight corresponds to a lower reliability index β_r , thereby necessitating additional material to attain the same level of reliability—resulting in an upward shift of the Pareto front. When compared with Figure 6, a noticeable upward displacement and slight clustering of the Pareto-optimal set can be observed due to the doubled standard deviation. This outcome underscores the importance of accounting for material and loading uncertainties (σ) in structural design: the greater the uncertainty, the higher the optimal weight required to ensure reliability. Consequently, designers must carefully balance safety requirements against material efficiency.

4. CONCLUSION

This study demonstrates the effectiveness of integrating the GDE3 algorithm with the IGA-FSDT model and FORM reliability analysis to solve a multi-objective optimization problem for multilayer composite plates under bending. By leveraging Pareto dominance and crowding distance mechanisms, GDE3 efficiently generates high-quality Pareto-optimal sets that balance weight reduction and deflection minimization. Furthermore, post-evaluation using FORM produces a reliable design, allowing designs to flexibly satisfy safety requirements. The robustness of the IGA–FSDT model was validated via a benchmark test case, with displacement errors kept below 1%, ensuring computational accuracy throughout the optimization process. The direct coupling between GDE3 and FORM automates the full “model-optimization-reliability assessment” loop, significantly reducing computational costs compared to decoupled approaches. These findings confirm that the GDE3–IGA/FSDT-FORM framework is a powerful tool for multi-objective laminate composite design. They also pave the way for future research integrating SORM or MCS methods to improve reliability precision, and adopting parallel computing strategies to accelerate the resolution of more complex industrial and aerospace design problems.

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