Transport and Communications Science Journal, Vol. 76, Issue 07 (09/2025), 951-964

Transport and Communications Science Journal

DESIGN OPTIMIZATION FOR MATERIAL REDUCTION IN MOTORCYCLE BRAKE DISCS

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ARTICLE INFO

TYPE: Research Article
Received: 16/07/2025
Revised: 09/08/2025
Accepted: 10/09/2025
Published online: 15/09/2025
https://doi.org/10.47869/tcsj.76.7.3
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Abstract. Brake discs are essential components in motorcycle braking systems, where structural strength and thermal stability significantly affect vehicle safety and performance. However, conventional brake disc designs often encounter trade-offs between weight reduction and mechanical durability. This study aims to analyze and optimize the design of a motorcycle brake disc to reduce weight and stress while maintaining structural integrity and performance. A standard disc model from the HONDA LEAD (SCR) was reconstructed and analyzed using Altair HyperMesh, integrating both static structural and modal analysis methods. Topology optimization via the OptiStruct module was applied to minimize material in low-stress regions. Following structural optimization, modal analysis was conducted to determine the natural frequencies and evaluate dynamic behavior under real-world conditions. The optimized design achieved a 9.4% reduction in mass and an 11.5% decrease in peak stress while maintaining the displacement and resonance frequencies within acceptable limits. These results confirm that the proposed approach significantly enhances brake disc performance and durability. This study demonstrates the effectiveness of combining finite element analysis and topology optimization in enhancing both static and dynamic characteristics of motorcycle brake components.

Keywords: Brake disc; Design optimization; CAE; Finite Element Method (FEM); HyperMesh; Topology optimization; Modal analysis.

1. INTRODUCTION

Design optimization plays a crucial role in mechanical engineering, where factors such as size, weight, strength, and cost must be carefully balanced to achieve an optimal design. Structural optimization, particularly topology and shape optimization, has been widely applied to refine components from the early stages of product development. These methods enable engineers to enhance strength and lifespan while minimizing material usage and overall weight. The integration of the Finite Element Method (FEM) further facilitates efficient problem-solving by reducing the need for extensive physical testing, lowering costs, and accelerating the design process [1]. Through computational analysis, an optimal design can be achieved based on predefined performance criteria, leading to safer and more durable mechanical components.

Among various mechanical systems, braking systems are essential for ensuring vehicle safety and performance. Modern motorcycles and cars typically employ a combination of disc and drum brakes, with disc brakes commonly installed on the front wheels due to their superior heat dissipation and braking efficiency. The effectiveness of a braking system is influenced by multiple factors, including material selection, design geometry, and operating conditions.

Numerous studies have initially focused on reducing brake disc mass while maintaining structural integrity. Topology optimization has emerged as a widely used method in this regard. For instance, A. Singh proposed a new brake disc design for off-road vehicles, utilizing topology optimization to achieve significant weight reduction without compromising stiffness [2]. Similarly, P. Sokolowski et al. focused on optimizing the mass of a brake disc for a lightweight three-wheeled vehicle, demonstrating its potential in improving fuel efficiency [3]. Meanwhile, S. Thigale and C. Shah applied the same optimization technique to reduce the weight of brake discs, further contributing to overall vehicle weight reduction [4]. In addition to these efforts, A. Oshinibosi et al. extended the scope of mass optimization by investigating how an optimized geometry could also enhance heat dissipation, thereby improving the thermal performance of the brake disc [5].

Beyond geometric optimization, material selection plays a crucial role in brake disc performance. Researchers have explored innovative materials to enhance durability, heat resistance, and overall efficiency. Notably, P. Kulkarni investigated composite materials for motorcycle brake discs, assessing their ability to replace conventional cast iron while maintaining strength and reducing weight [6]. Likewise, Alnaqi et al. analyzed functionally graded materials (FGM) in brake discs, highlighting their potential in minimizing thermal stress and increasing component lifespan [7]. These findings suggest that material innovation could significantly improve braking performance and durability.

In addition to structural and material considerations, heat management remains a critical aspect of brake disc design. Several studies have focused on understanding and optimizing temperature distribution and thermal loads. For example, Tang et al. examined ventilated brake discs, analyzing the influence of ventilation structures on heat dissipation and load distribution [8]. Meanwhile, Chavan et al. employed computational fluid dynamics (CFD) simulations to evaluate the cooling performance of ventilated brake discs, identifying design improvements to optimize airflow [9]. Similarly, Bhat et al. utilized CFD simulations to further investigate airflow dynamics through brake discs, offering valuable insights into heat exchange mechanisms [10]. Furthermore, the review study by Dewanto et al. provided a comprehensive evaluation of various brake disc designs, discussing key factors influencing heat dissipation performance and durability [11].

Lastly, in addition to studies focusing on design improvements, some research has explored friction material performance and overall brake disc evaluations. Tang et al. analyzed different friction materials under dry and wet braking conditions, providing an overview of their performance and durability [12]. Meanwhile, Dewanto et al. compiled an extensive review of brake disc designs, assessing optimization methods and highlighting future research directions [11].

While previous studies have focused on individual aspects of material, geometry, or cooling systems, this research proposes a comprehensive optimization approach for Honda Lead (SCR) motorcycle brake discs through integrated static structural analysis, topology optimization, and modal analysis. To achieve this, we employ the Finite Element Method (FEM) to analyse stress distribution and deformation under braking conditions. The optimization process aims to reduce both weight and stress, thereby improving vehicle efficiency and extending the lifespan of the braking system. Following the structural optimization, a Modal Analysis is conducted to determine the natural frequencies of the brake disc, ensuring that resonance effects are minimized. Unlike conventional optimization methods that often result in overly complex geometries, this study prioritizes manufacturing practicality by employing design solutions adaptable to standard production techniques. The proposed methodology seeks to achieve an optimal balance between weight reduction and structural integrity, ensuring reliable performance in actual operating environments.

2. METHODOLOGY

2.1. Design of existing models

The brake disc model of the Honda Lead (SCR) was developed based on actual measurements and available standard values. The brake disc was precisely measured and redesigned using Autodesk Inventor software to ensure conformity with the original technical specifications.

2.2. Evaluation of existing models

Structural analysis, optimization, and modal analysis performed using Altair HyperMesh are critical steps in the design and improvement of brake discs [13]. In addition to evaluating stress and deformation evaluation under applied loads and boundary conditions, material selection plays a vital role. Based on standard references and manufacturer specifications, S50C steel was selected as the brake disc material, representing the standard choice for motorcycle brake discs.

However, material quality cannot be determined solely by chemical composition. Foundries may adjust these components provided the mechanical properties meet required standards.

In structural analysis, the deformation and stress of the material under the influence of boundary conditions are studied. Factors that apply force to the brake disc, such as braking torque, clamping force and the contact area of the brake pads, are used as input boundary conditions for the problem. After applying these boundary conditions, the structural analysis will provide results including overall deformation, von-Mises stress, maximum stress, displacement, etc…, which have been calculated and evaluated to determine the effectiveness of the design [1].

Table 1. The chemical composition of S50C steel.

Composition	(%)
Carbon (C)	0.5
Silicon (Si)	0.17 - 0.37
Manganese (Mn)	0.6 - 0.9
Phosphorus (P)	≤ 0.035
Sulfur (S)	≤ 0.035%

Table 2. Mechanical Properties of S50C Steel

Young's Modulus	210	GPa
Tensile Strength	550	MPa
Yield Strength	355	MPa
Poisson's ratio	0.3	

During the optimization process, design factors such as shape, material and structure are adjusted to achieve maximum performance, with the goal of reducing weight, stress, and manufacturing costs while still meeting technical requirements. This helps fine-tune the design parameters to improve load-bearing capacity, durability and overall performance of the product [14], [15].

For modal analysis, natural frequencies and mode shapes are identified to prevent resonance, which could compromise the brake disc's strength and performance [16].

3. FIGURES AND TABLES

3.1. Initial conditions and assumptions

To ensure accuracy in calculations, the initial condition is taken from the dealer's site, and the relevant assumptions are made as follows.

- The total weight of the vehicle is assumed to be 274kg.

- The vehicle is assumed to travel at a maximum speed of 120km/h, v = 33.33m/s.

- According to Circular 31/2019/TT-BGTVT [18], for speeds of $100 < V \leq 120$ (km/h), the minimum stopping distance will be: 100m.

- The axial weight distribution is taken as 0.5.

- The coefficient of friction is assumed to be 0.5.

- The effective radius is taken as, r eff = 0.095 m.

- The standard hydraulic pressure is taken as 1 MPa

- The coefficient of friction is same for brake pad and rotor, i.e., $\mu I = \mu O$.

- The brake pad's total coverage angle is measured to be 45°.

- The tangential clamping force between the brake pad and rotor on inside is equal to outside, i.e., FTRI = FTRO, FRI = FRO.

- The vehicle is said to stop using one brake caliper, i.e., the stopping distance is taken as 100 meters.

3.2. Calculations

To ensure that simulation results accurately reflect real operating conditions, it is essential to determine the external forces acting on the brake disc. These include the normal force applied by the brake pads, the resulting tangential (friction) forces, and the total clamping force during braking. Accurate calculation of these forces provides the necessary input for stress analysis and torque estimation, both of which directly affect the structural performance of the disc. The following equations (Eqs. 1–8) describe the mathematical formulations used to compute the contact area, normal force, tangential force, clamping force, and braking torque. These formulas are based on mechanical principles of disc brakes and are consistent with widely accepted references in the field of vehicle dynamics and brake system analysis [1],[17].

- The contact area of the brake pad is calculated as follows (Eq. 1):

$$A = \pi(r_1^2 - r_2^2)\frac{\theta}{360} = 0.00198\ \text{m}^2 \tag{1}$$

- Norm force on inside (Eq. 2):

$$F_{RI} = \left(\frac{P_{\max}}{2}\right)A = 900N \tag{2}$$

- Tang reacts force on inside (Eq. 3):

$$F_{TRI} = \mu_I F_{RI} = 495N \tag{3}$$

- Tang reacts force on outside (Eq. 4):

$$F_{TR0} = \mu_o F_{Ro} = 495N \tag{4}$$

- Tang clamping force (Eq. 5):

$$F_T = F_{TRI} + F_{TR0} = 990N \tag{5}$$

- Brake torque (Eq. 6):

$$TB = F_T r = 94Nm \tag{6}$$

- The equation of linear velocity in motion is [17] (Eq. 7):

$$v^2 = v_0^2 + 2as \tag{7}$$

Where:

a: Acceleration of motion (m/s^2)

v: Final velocity of the vehicle (m/s)

v_0: Initial velocity of the vehicle before braking begins (m/s)

s: Displacement (m)

From Eq.7, the acceleration of motion can be calculated:

$$a = \frac{v^2 - v_o^2}{2S} = -5.55(m/s^2)$$

- The linear velocity-time relation [17] (Eq. 8):

$$v = v_0 + at \tag{8}$$

From Eq. (8), the braking time is calculated as:

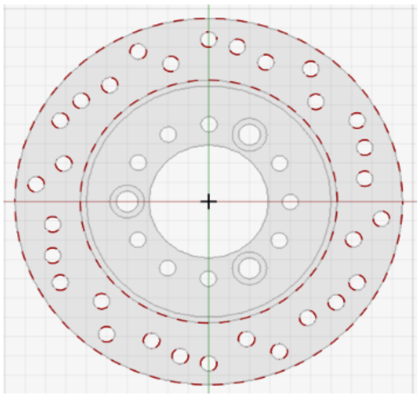
$$t = \frac{v - v_0}{a} = \frac{0 - 33.33}{-5.55} = 6 \text{ (s)}$$

3.3. The design of disc brake rotors

The brake disc model used in this study is based on the standard brake disc of the Honda Lead (SCR), as shown in Figure 1a. The disc was measured and reconstructed into a 3D model, as illustrated in Figure 1b. This redesigned model serves as the foundation for further analysis and optimization, ensuring accuracy in simulating real-world conditions.

Conventionally, in mathematical equations variables and anything that represents a value appear in italics. You may choose to number equations for easy referencing. In that case the number should appear at the right margin.

a) Standard brake disc model of the Honda Lead (SCR)



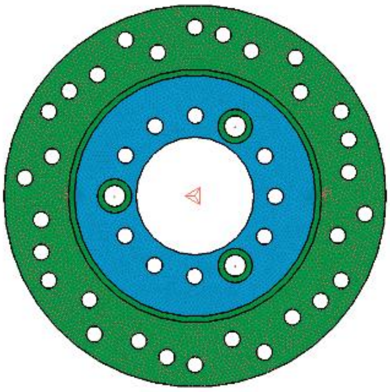
b) Reconstructed 3D CAD model

Figure 1. Standard and designed brake disc models.

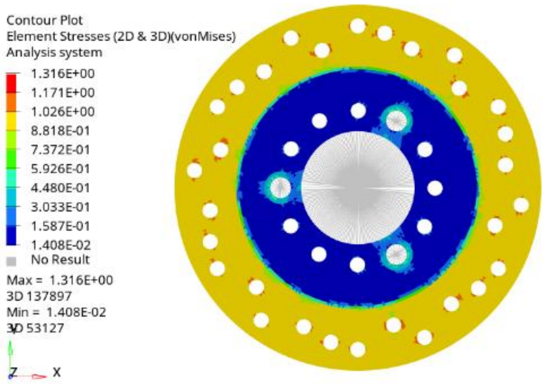
4. ANALYSIS AND OPTIMIZATION OF DISC BRAKE ROTORS

For the optimization analysis of the brake disc, the following input parameters were defined:

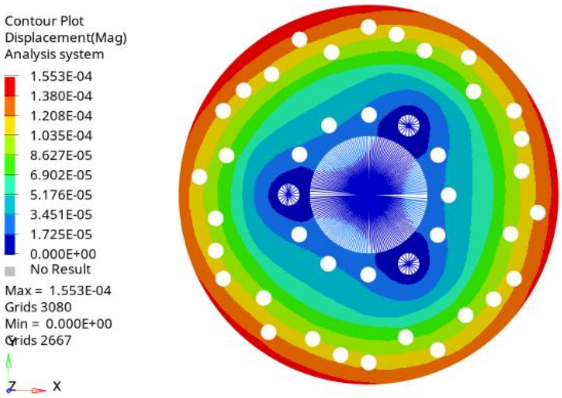
- Import the 3D CAD model of the standard brake disc into HyperMesh, clean the geometry, and generate a mesh using second-order tetrahedral elements
- Define the design space, excluding non-design regions such as the hub mounting area and bolt holes.
- The material S50C steel was selected, with a Young's modulus of 205 GPa, Poisson's ratio of 0.29, and density of 7850 kg/m³
- Apply boundary conditions by fully constraining the three inner bolt holes to replicate the actual mounting constraints.
- Apply loading conditions, including a brake torque of 94 Nm and a uniform pressure of 1 MPa on the braking surface
- Set up the topology optimization in OptiStruct with the objective of minimizing mass, constraining von Mises stress and displacement to allowable limits
- Use the SIMP method with a density threshold to distinguish material and void, and apply symmetry constraints for manufacturability.
- Perform post-processing to interpret the density results, reconstruct and smooth the optimized shape, and validate it through static and modal analyses.



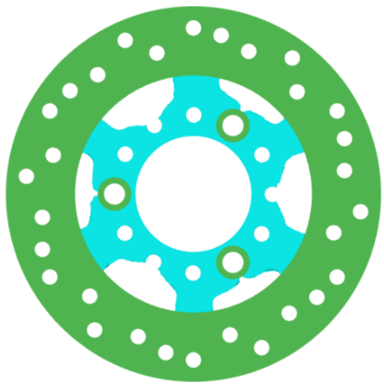
a) 3D model with boundary conditions



b) Stress of motorcycle brake disc model



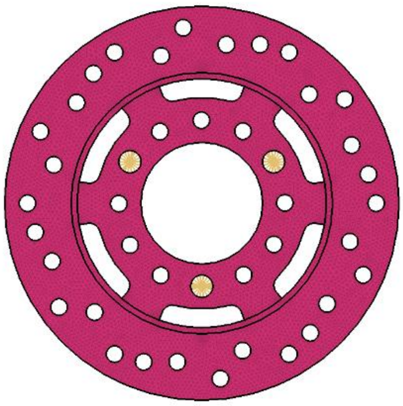
c). Displacement of motorcycle brake disc model



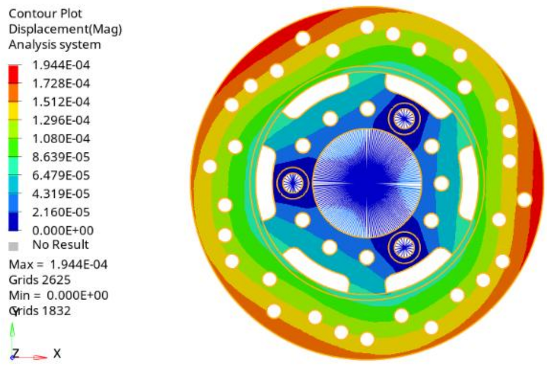
d) Optimization model of brake disc

Figure 2. Simulation process and results for motorcycle brake disc.

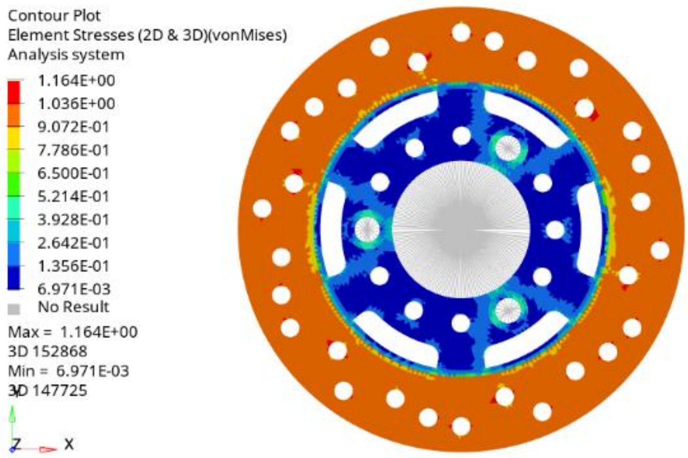
The optimization process aimed to reduce the weight of the brake disc while ensuring the von Mises stress remained below the yield strength of S50C steel (355 MPa). The initial setup (Figure 2a) shows the model with boundary conditions applied. Stress analysis (Figure 2b) indicates a maximum von Mises stress of approximately 1.316 MPa, mainly concentrated around the bolt holes and inner contact surfaces. Displacement results (Figure 2c) show a peak value of 1.535×10^{-4} mm, occurring at the outer edge of the disc. The optimized topology (Figure 2d) clearly identifies low-stress and low-displacement regions—particularly in the outer and intermediate areas—that can be safely removed to reduce weight while maintaining structural integrity. The abstract optimization results were transformed into a practical, manufacturable brake disc geometry (Figure 3a), preserving key structural features from the original design. Displacement analysis of the redesigned model (Figure 3b) shows a maximum value of 1.092×10^{-4} mm, reduced compared to the original model (1.535×10^{-4} mm), indicating improved stiffness. Similarly, the von Mises stress distribution (Figure 3c) remains within safe limits, with a maximum of 1.283 MPa, slightly lower than the initial 1.316 MPa, and redistributed more evenly. These findings validate the effectiveness of the optimization, demonstrating enhanced mechanical performance while maintaining manufacturability.



a) Redesign of the brake disc model after the optimization process



b) Displacement of the brake disc model after the optimization process



c) Stress of the brake disc model after the optimization process

Figure 3. Post-optimization results of the motorcycle brake disc.

5. EVALUATION OF OPTIMIZATION RESULTS

The optimization process delivered measurable improvements in the brake disc's structural performance, as evidenced by both visual results (Figures 2–3) and quantitative data (Table 2). The redesigned brake disc achieved a 9.4% weight reduction, decreasing from 0.54 kg to 0.49 kg (Figures 3a), which contributes to improved vehicle acceleration and fuel efficiency. Furthermore, the maximum von Mises stress decreased by 11.5%, from 1.31 MPa in the initial design (Figure 2b) to 1.16 MPa in the optimized model (Figure 3c), enhancing fatigue resistance and extending service life. While the maximum displacement slightly increased from 0.15 mm to 0.19 mm (Figure 3b), the value remained well within acceptable engineering tolerances, ensuring overall structural integrity. These results confirm that the optimization successfully achieved its objectives—reducing both weight and stress while maintaining durability and safety.

Table 3. The results of the brake rotor and the brake rotor after optimization

	Preliminary Design	Design after Optimization
Maximum Stress (MPa)	1.31	1.16
Maximum Displacement (mm)	0.15	0.19
Mass (kg)	0.54	0.49 $(\approx 9.4\%)$

6. MODAL ANALYSIS OF DISC BRAKE ROTORS

6.1. Brake disc constraints

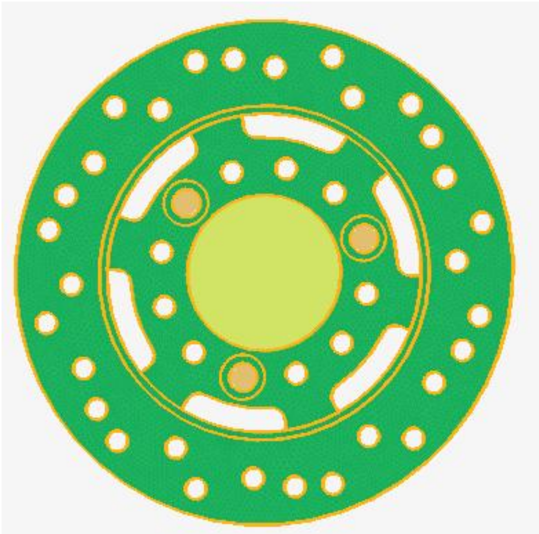
In a constrained Modal Analysis problem, the natural frequency and vibration mode of the brake disc in Mode 1 are the most critical factors. Resonance occurs when both the vibration direction and frequency align, leading to increased vibration amplitude, which can result in uneven wear and structural damage. Mode 1, being the mode with the lowest frequency, represents the fundamental vibrational behavior of the system, making it essential to consider in the design and analysis process.

These conditions and the simulation results are illustrated in Figure 4, showing both the applied constraints (Figure 4a) and the vibration shape with its corresponding natural frequency in Mode 1 (Figure 4b).

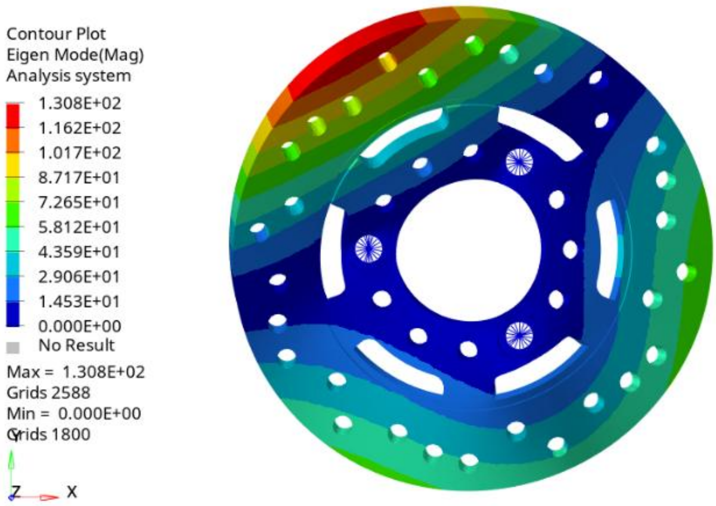
For the constrained Modal Analysis of the brake disc, the following input settings were applied:

- Constraints: The three inner bolt holes were fixed to simulate the actual mounting conditions.
- Material: S50C steel was used, ensuring appropriate mechanical properties for the analysis.
- Mesh: A preferred element size was selected to balance both accuracy and computational efficiency.

As shown in Figure 4b, the vibration mode shape in Mode 1 is characterized by a symmetrical out-of-plane deformation, primarily concentrated at the outer rim of the brake disc and around the ventilation holes. These are regions with lower stiffness, making them more susceptible to dynamic response. The natural frequency of 130.8 Hz is relatively low, indicating that this mode represents the fundamental dynamic behavior of the system. The absence of high deformation near the bolt holes confirms the effectiveness of the boundary constraints, while the concentration of displacement at the free edges suggests potential areas for structural reinforcement or mass reduction if needed.



a) Model under applied boundary conditions

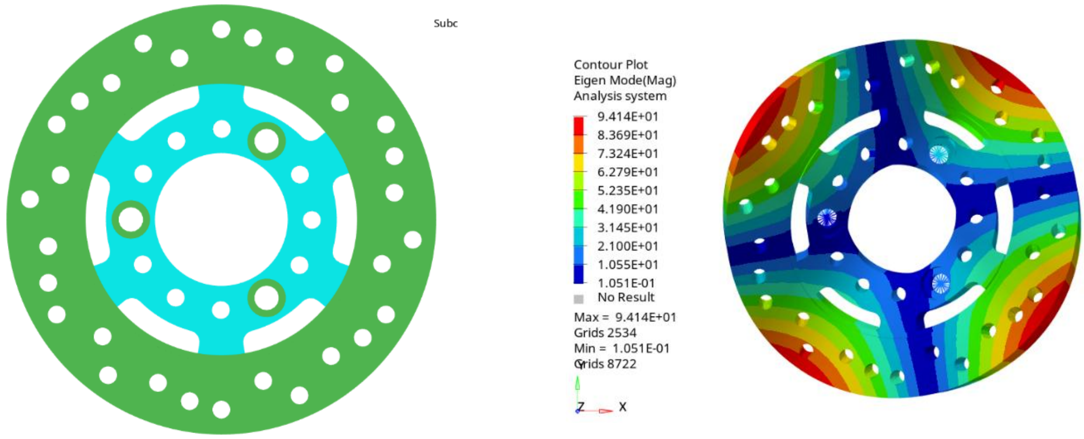


b) Vibration shape and natural frequency of the brake disc in Mode 1

Figure 4. Results of the constrained modal analysis of the brake disc

6.2. Brake disc without constraints

For the modal analysis problem without constraints, the first six vibration modes typically exhibit very low frequencies—almost approaching zero—as they correspond to the system's six rigid-body degrees of freedom (three translational and three rotational). Therefore, these modes are disregarded, and analysis begins from Mode 7 onwards to eliminate insignificant vibrations and focus on meaningful dynamic responses. By examining the vibration modes from Mode 7 and beyond, we can identify the areas where deformation energy is concentrated, which helps reveal structurally weak zones. These insights are critical for guiding design improvements that enhance both durability and stability, thereby improving overall structural performance. The results of this analysis are shown in Figure 5, where the deformation pattern and natural frequency of Mode 7 are visualized.



a) Model without applied boundary conditions

b) Vibration shape and natural frequency of the brake disc in Mode 7

Figure 5. Results of the unconstrained modal analysis of the brake disc.

Table 3. The table of natural frequencies of the constrained brake rotor and the unconstrained brake rotor

Mode	Natural frequency (Hz) (With constraints)	Natural frequency (Hz) (Without constraints)
1	475	0
2	476	0
3	511	0
4	749	0
5	750	0
6	1247	0
7	1291	486
8	2150	487
9	2154	776
10	2660	1209

7. CONCLUSION

This study focused on optimizing the design of a motorcycle brake disc through static analysis, structural optimization, and modal analysis using Altair HyperMesh software. The process began with redesigning the HONDA LEAD (SCR) brake disc based on actual measurements, using S50C steel as the material. Static analysis was conducted to evaluate stress and deformation under braking forces and pressure. Subsequently, optimization techniques were applied to reduce both weight and stress while ensuring sufficient structural strength. Finally, modal analysis was performed to determine natural frequencies and avoid resonance.

The optimization process yielded significant improvements in both structural performance and weight reduction. Notably, the redesigned brake disc demonstrated a 9.4% mass reduction, decreasing from 0.54 kg to 0.49 kg, which contributes to enhanced fuel efficiency and vehicle dynamics. Equally important, the maximum stress was reduced by 11.5%, dropping from 1.31 MPa to 1.16 MPa, thereby improving fatigue resistance and extending the component's service life. Although the maximum displacement increased marginally from 0.15 mm to 0.19 mm, it remained well within safe operational limits. Furthermore, modal analysis provided critical insights into the disc's vibrational behavior, identifying key natural frequencies such as 475 Hz (constrained Mode 1) and 486 Hz (unconstrained Mode 7). These findings are instrumental in mitigating resonance-induced mechanical failures, ensuring reliable braking performance under real-world conditions.

While this study was intentionally focused on optimizing the brake disc design for weight reduction through static and modal analyses under linear assumptions—directly addressing the goal of reducing rotational inertia in motorcycles to improve fuel efficiency, acceleration, and handling—several important aspects remain beyond its current scope. In our future research, we will incorporate thermal and thermo-mechanical analyses to assess braking heat effects, conduct fatigue analysis to estimate service life under high-cycle loading, and develop nonlinear models to capture contact slip, large deformation, and plasticity under extreme conditions. We will also investigate advanced materials such as carbon-ceramic composites, integrate multidisciplinary optimization approaches, validate results through dynamometer testing, and explore AI-driven generative design to create innovative geometries with enhanced structural and thermal performance. The practical feasibility of such design improvements is demonstrated by the two manufactured models—before and after optimization—shown in Figure 6a–b, highlighting the successful translation of simulation results into real-world applications.

a) Original brake disc *b) Brake disc after optimization*

Figure 6. Physical models of the brake disc before and after optimization.

ACKNOWLEDGMENT

This research is funded by University of Transport and Communications (UTC) under grant number T2025-CK-012.

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