



## EVALUATION OF LIMITING PROBABILITIES WHEN TRANSITIONING STATES OF SUBSYSTEMS IN METRO TRAINS OPERATING ON THE CAT LINH - HA DONG LINE

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**Abstract.** The rolling stock on the Cat Linh - Ha Dong urban railway line, as an independent technical system, is divided into six subsystems: body, running gears, transmission, braking equipment, control, and auxiliary equipment. Reliability evaluation of rolling stock includes many contents such as assessing the reliability of subsystems, cars, and trains; reliability and lifetime of wheelsets due to wear and tear; limiting probabilities when transitioning states of subsystems and rolling stock. The article presents the evaluation of limiting probabilities when transitioning states of subsystems on metro trains in operation on the Cat Linh - Ha Dong line by using the software “Evaluation of the reliability and safety of rolling stock in operation on the Cat Linh - Ha Dong urban railway line”. The calculation results show the most critical elements and subsystems within a specified operational time period, helping users have appropriate solutions in planning spare parts, and replacement materials, reducing downtime for maintenance and repair, and improving the reliability, availability and operating efficiency of rolling stock.

**Keywords:** limiting probability, state transition, metro trains, Cat Linh - Ha Dong, operation.

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## 1. INTRODUCTION

Rolling stock in urban rail systems (metro trains), as independent complex technical systems, is divided into subsystems such as the body, running gear, transmission, braking equipment, control, and auxiliary equipment. To evaluate the reliability of rolling stock, it is necessary to assess the reliability of its subsystems [1, 2]. The metro trains operating on the Cat Linh - Ha Dong line are classified into six subsystems, each of which is further divided into sub-subsystems, and each sub-subsystem consists of individual elements [2-5]. Specifically: 1. The Body subsystem is divided into six sub-subsystems with a total of 11 elements; 2. The Running Gear subsystem consists of one sub-subsystem and 13 elements; 3. The Transmission subsystem includes one sub-subsystem and 10 elements; 4. The Braking Equipment subsystem comprises one sub-subsystem and 15 elements; 5. The Control subsystem consists of one sub-subsystem and 10 elements; 6. The Auxiliary Equipment subsystem is divided into six sub-subsystems with a total of 30 elements. Thus, the entire system consists of six subsystems, 16 sub-subsystems, and 89 elements.

Based on statistical data on failures occurring during train operation, including failures that caused service disruptions and those requiring unscheduled maintenance or repairs at the depot, from November 6, 2021, to March 31, 2024, the software, developed by the authors [6], has been utilized to identify and evaluate the reliability of the entire rolling stock over this period. This article presents the evaluation results of the limiting probabilities when transitioning states of subsystems on metro trains in operation on the Cat Linh - Ha Dong line.

## 2. BASIS FOR EVALUATING THE LIMITING PROBABILITIES WHEN TRANSITIONING STATES OF SUBSYSTEMS IN THE ROLLING STOCK OPERATING ON THE CAT LINH - HA DONG URBAN RAILWAY LINE

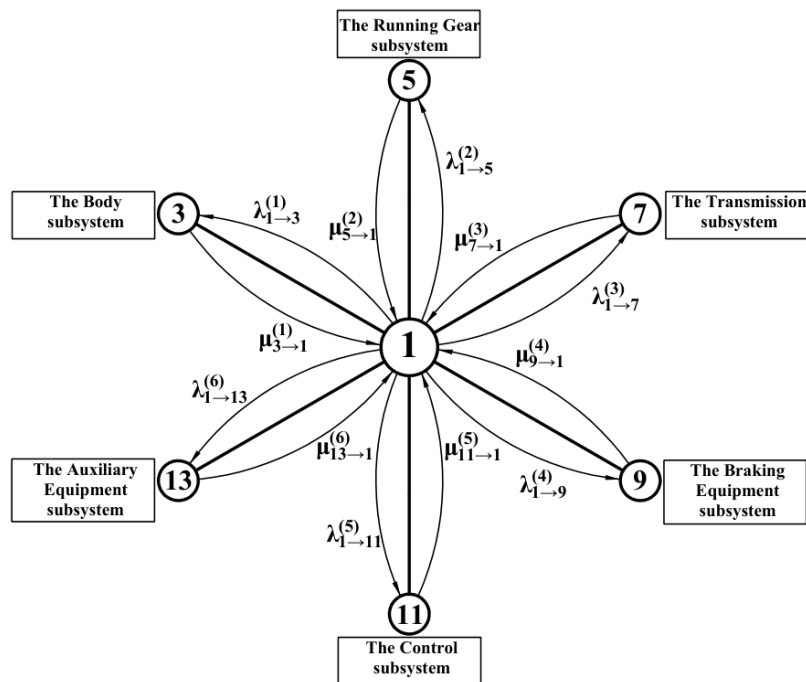


Figure 1. The limiting transition states model of the rolling stock on the Cat Linh - Ha Dong line with 6 subsystems.

Based on a synthesis of literature concerning the reliability assessment of various systems and components across multiple engineering domains - including mechanical engineering, electric power systems, electronics, and thermal energy [6-12] - a specialized computational model has been developed for urban railway rolling stock. This model includes a probabilistic framework for estimating the limiting probabilities when transitioning states of subsystems on metro trains in operation on the Cat Linh - Ha Dong line, as illustrated in Fig. 1 [1, 2].

The state transition limit matrix of the rolling stock on the Cat Linh - Ha Dong line, consisting of 6 subsystems, is presented in Table 1 [1, 2].

Table 1. The state transition limit matrix of the rolling stock on the Cat Linh - Ha Dong line with 6 subsystems.

$-\sum_{j=3,5,\dots,13} \lambda_{1 \rightarrow j}$	$\lambda_{1 \rightarrow 3}^{(1)}$	$\lambda_{1 \rightarrow 5}^{(2)}$	$\lambda_{1 \rightarrow 7}^{(3)}$	$\lambda_{1 \rightarrow 9}^{(4)}$	$\lambda_{1 \rightarrow 11}^{(5)}$	$\lambda_{1 \rightarrow 13}^{(6)}$
$\mu_{3 \rightarrow 1}^{(1)}$	$-\mu_{3 \rightarrow 1}^{(1)}$	0	0	0	0	0
$\mu_{5 \rightarrow 1}^{(2)}$	0	$-\mu_{5 \rightarrow 1}^{(2)}$	0	0	0	0
$\mu_{7 \rightarrow 1}^{(3)}$	0	0	$-\mu_{7 \rightarrow 1}^{(3)}$	0	0	0
$\mu_{9 \rightarrow 1}^{(4)}$	0	0	0	$-\mu_{9 \rightarrow 1}^{(4)}$	0	0
$\mu_{11 \rightarrow 1}^{(5)}$	0	0	0	0	$-\mu_{11 \rightarrow 1}^{(5)}$	0
$\mu_{13 \rightarrow 1}^{(6)}$	0	0	0	0	0	$-\mu_{13 \rightarrow 1}^{(6)}$

The system of state transition limit equations for the urban railway rolling stock on the Cat Linh - Ha Dong line, consisting of six subsystems, is expressed in Eq. (1) [1, 2].

$$\left. \begin{aligned} & - \left( \sum_{j=3,5,\dots,13} \lambda_{1 \rightarrow j} \right) P_1 + \mu_{3 \rightarrow 1}^{(1)} P_3^{(1)} + \mu_{5 \rightarrow 1}^{(2)} P_5^{(2)} + \mu_{7 \rightarrow 1}^{(3)} P_7^{(3)} + \mu_{9 \rightarrow 1}^{(4)} P_9^{(4)} + \mu_{11 \rightarrow 1}^{(5)} P_{11}^{(5)} + \mu_{13 \rightarrow 1}^{(6)} P_{13}^{(6)} = 0 \\ & \lambda_{1 \rightarrow 3}^{(1)} P_1 - \mu_{3 \rightarrow 1}^{(1)} P_3^{(1)} = 0 \\ & \lambda_{1 \rightarrow 5}^{(2)} P_1 - \mu_{5 \rightarrow 1}^{(2)} P_5^{(2)} = 0 \\ & \lambda_{1 \rightarrow 7}^{(3)} P_1 - \mu_{7 \rightarrow 1}^{(3)} P_7^{(3)} = 0 \\ & \lambda_{1 \rightarrow 9}^{(4)} P_1 - \mu_{9 \rightarrow 1}^{(4)} P_9^{(4)} = 0 \\ & \lambda_{1 \rightarrow 11}^{(5)} P_1 - \mu_{11 \rightarrow 1}^{(5)} P_{11}^{(5)} = 0 \\ & \lambda_{1 \rightarrow 13}^{(6)} P_1 - \mu_{13 \rightarrow 1}^{(6)} P_{13}^{(6)} = 0 \end{aligned} \right\} (1)$$

Where:

### 1. For Subsystem 1 (Body):

$\lambda_{1 \rightarrow 3}^{PH1} = \lambda_{1 \rightarrow 3}^{(1)}$  - The failure rate of Subsystem 1 (Body) represents the transition from State 1 (Operating - LV) to State 3 (Complete Failure - H<sub>ht</sub>);

$\mu_{3 \rightarrow 1}^{PH1} = \mu_{3 \rightarrow 1}^{(1)}$  - The recovery rate of Subsystem 1 (Body) represents the transition from State 3 (H<sub>ht</sub>) to State 1 (LV);

$P_3^{(1)}$  - The probability of the rolling stock being in State 3, a non-operational state due to the failure of Subsystem 1 (Body).

## 2. For Subsystem 2 (Running Gear):

$\lambda_{1 \rightarrow 5}^{PH2} = \lambda_{1 \rightarrow 5}^{(2)}$  - The failure rate of Subsystem 2 (Running Gear) represents the transition from State 1 (LV) to State 5 (H<sub>ht</sub>);

$\mu_{5 \rightarrow 1}^{PH2} = \mu_{5 \rightarrow 1}^{(2)}$  - The recovery rate of Subsystem 2 (Running Gear) represents the transition from State 5 (H<sub>ht</sub>) to State 1 (LV);

$P_5^{(2)}$  - The probability of the rolling stock being in State 5, a non-operational state due to the failure of Subsystem 2 (Running Gear).

## 3. For Subsystem 3 (Transmission):

$\lambda_{1 \rightarrow 7}^{PH3} = \lambda_{1 \rightarrow 7}^{(3)}$  - The failure rate of Subsystem 3 (Transmission) represents the transition from State 1 (LV) to State 7 (H<sub>ht</sub>).;

$\mu_{7 \rightarrow 1}^{PH3} = \mu_{7 \rightarrow 1}^{(3)}$  - The recovery rate of Subsystem 3 (Transmission) represents the transition from State 7 (H<sub>ht</sub>) to State 1 (LV);

$P_7^{(3)}$  - The probability of the rolling stock being in State 7, a non-operational state due to the failure of Subsystem 3 (Transmission).

## 4. For Subsystem 4 (Braking Equipment):

$\lambda_{1 \rightarrow 9}^{PH4} = \lambda_{1 \rightarrow 9}^{(4)}$  - The failure rate of Subsystem 4 (Braking Equipment) represents the transition from State 1 (LV) to State 9 (H<sub>ht</sub>);

$\mu_{9 \rightarrow 1}^{PH4} = \mu_{9 \rightarrow 1}^{(4)}$  - The recovery rate of Subsystem 4 (Braking Equipment) represents the transition from State 9 (H<sub>ht</sub>) to State 1 (LV);

$P_9^{(4)}$  - The probability of the rolling stock being in State 9, a non-operational state due to the failure of Subsystem 4 (Braking Equipment).

## 5. For Subsystem 5 (Control):

$\lambda_{1 \rightarrow 11}^{PH5} = \lambda_{1 \rightarrow 11}^{(5)}$  - The failure rate of Subsystem 5 (Control) represents the transition from State 1 (LV) to State 11 (H<sub>ht</sub>);

$\mu_{11 \rightarrow 1}^{PH5} = \mu_{11 \rightarrow 1}^{(5)}$  - The recovery rate of Subsystem 5 (Control) represents the transition from State 11 (H<sub>ht</sub>) to State 1 (LV);

$P_{11}^{(5)}$  - The probability of the rolling stock being in State 11, a non-operational state due to the failure of Subsystem 5 (Control).

## 6. For Subsystem 6 (Auxiliary Equipment):

$\lambda_{1 \rightarrow 13}^{PH6} = \lambda_{1 \rightarrow 13}^{(6)}$  - The failure rate of Subsystem 6 (Auxiliary Equipment) represents the transition from State 1 (LV) to State 13 (H<sub>ht</sub>);

$\mu_{13 \rightarrow 1}^{PH6} = \mu_{13 \rightarrow 1}^{(6)}$  - The recovery rate of Subsystem 6 (Auxiliary Equipment) represents the transition from State 13 (H<sub>ht</sub>) to State 1 (LV);

$P_{13}^{(6)}$  - The probability of the rolling stock being in State 13, a non-operational state due to the failure of Subsystem 6 (Auxiliary Equipment).

### 7. For the Overall System - Rolling Stock:

$P_1$  - The probability of the rolling stock being in State 1, the operational state (representing the reliability of the overall system - rolling stock).

By solving the linear system of Eq. (1) with unknowns  $P_i$ ,  $i = 1, 3, 5, 7, 9, 11, 13$  and  $\sum_{i=1}^{13} P_i = 1$ , then applying the appropriate conditions along with their respective values  $\lambda_{i \rightarrow j}^{(k)}$  and  $\mu_{j \rightarrow i}^{(k)}$ , the limit probabilities or system reliability metrics can be determined.

## 3. EVALUATION OF LIMITING PROBABILITIES WHEN TRANSITIONING STATES OF SUBSYSTEMS ON THE ROLLING STOCK OPERATING ON THE CAT LINH - HA DONG URBAN RAILWAY LINE

Based on the Java programming language [13-15] and the SQLite database management system [16-19], the software “Evaluation of the reliability and safety of rolling stock in operation on the Cat Linh - Ha Dong urban railway line” [20] has been developed with the following general functions:

1. Evaluating the reliability and availability of the rolling stock during operation on the line.
2. Assessing the availability and maintainability of the rolling stock during maintenance and repair at the depot.
3. Evaluating the reliability and service life of wheelsets due to wear during operation on the line.
4. Assessing the safety of the rolling stock during operation on the line.
5. Evaluating the limiting probabilities when transitioning states of the rolling stock during operation on the line.

During the period from the start of operation on November 6, 2021, to March 31, 2024 (nearly 29 months), statistical data on failures of metro trains operating on the Cat Linh - Ha Dong line have been collected, as summarized in Table 2.

Table 2. Statistical data on failures of metro rolling stock on the Cat Linh - Ha Dong line.

On the line		At the depot	Wheel wear		
Failures causing train operation disruptions (delays)	Failures compromising safety and accidents	Failures leading to unscheduled repairs	Data on tread wear	Data on flange wear	Data on flange height increase
01	0	264	416	416	416

Among the 264 failures that resulted in unscheduled repairs at the depot, the distribution across subsystems is as follows: (1) Body subsystem: 33 failures; (2) Running Gear subsystem: 0 failures; (3) Transmission subsystem: 46 failures; (4) Braking Equipment subsystem: 61

failures; (5) Control subsystem: 3 failures; and (6) Auxiliary Equipment subsystem: 121 failures.

Using the aforementioned software, the reliability of the entire rolling stock has been identified and evaluated over the surveyed period, corresponding to the specified functions. The computational results are extensive, and due to the limited scope of a single article, it is not possible to present all findings in one publication. Instead, these results will be discussed across multiple articles. Therefore, this article focuses solely on presenting the evaluation results of the limiting probabilities when transitioning states of subsystems in metro trains operating on the Cat Linh - Ha Dong line, both during line operation and during maintenance and repair at the depot.

### 3.1. Evaluation of the limiting probabilities when transitioning states of subsystems on metro trains operating on the Cat Linh - Ha Dong line

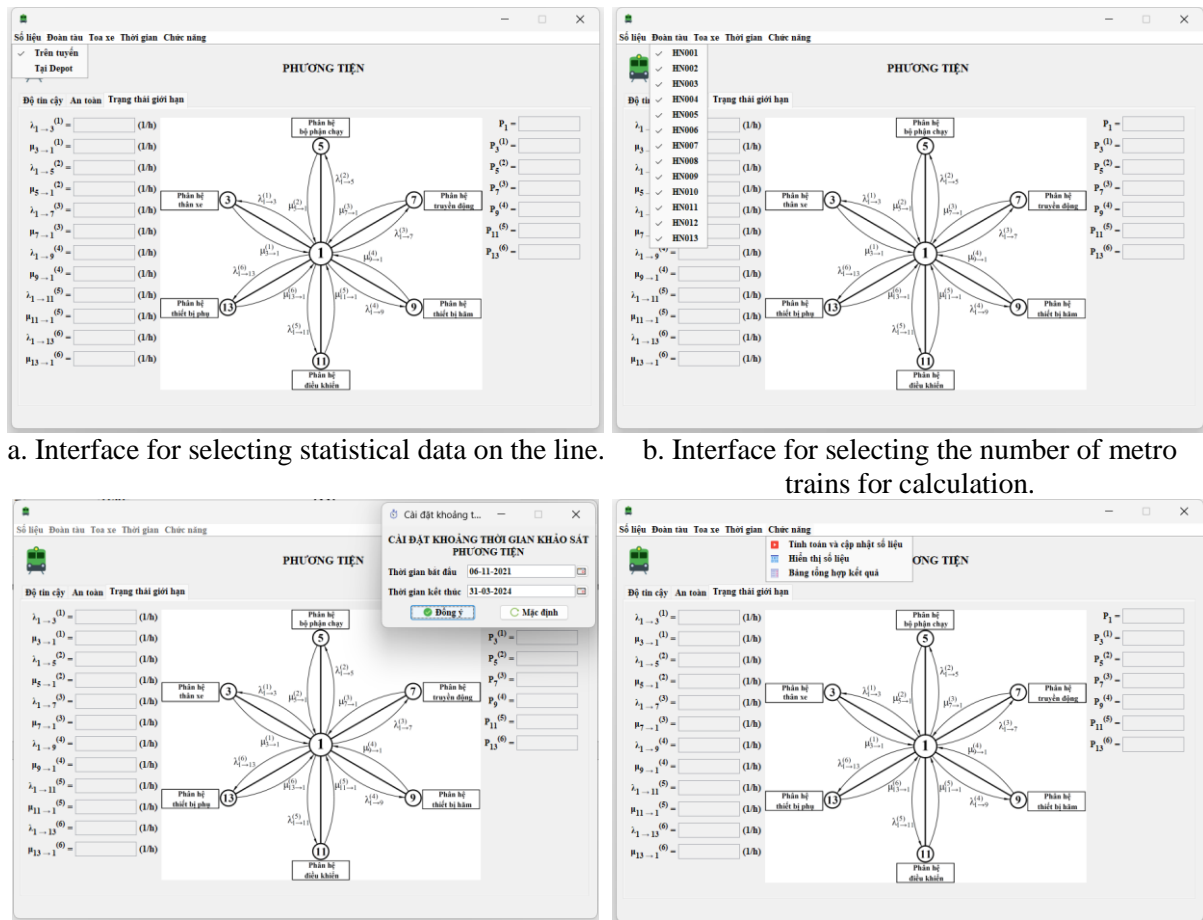


Figure 2. Interfaces for selecting calculation functions for the limiting probabilities when transitioning states of subsystems in the rolling stock during operation on the line.

As shown in Table 1, only one failure occurred on the line: on August 4, 2023, metro train HN008, specifically car Tc2, experienced a malfunction in the Auxiliary Equipment subsystem, within the Cab Electrical sub-subsystem, affecting the Control Cabinet element. The train was

unable to depart from Cat Linh Station due to a loss of traction caused by a 5 km/h 20K72 relay failure, resulting in a five-minute delay before train HN013 was dispatched as a replacement.

Thus, during the observed period, the reliability of the entire rolling stock depended solely on the Control Cabinet element within the Cab Electrical sub-subsystem of the Auxiliary Equipment subsystem.

The process of calculating and evaluating the limiting probabilities when transitioning states of subsystems on metro trains operating on the Cat Linh - Ha Dong line is illustrated in Fig. 2 and 3.

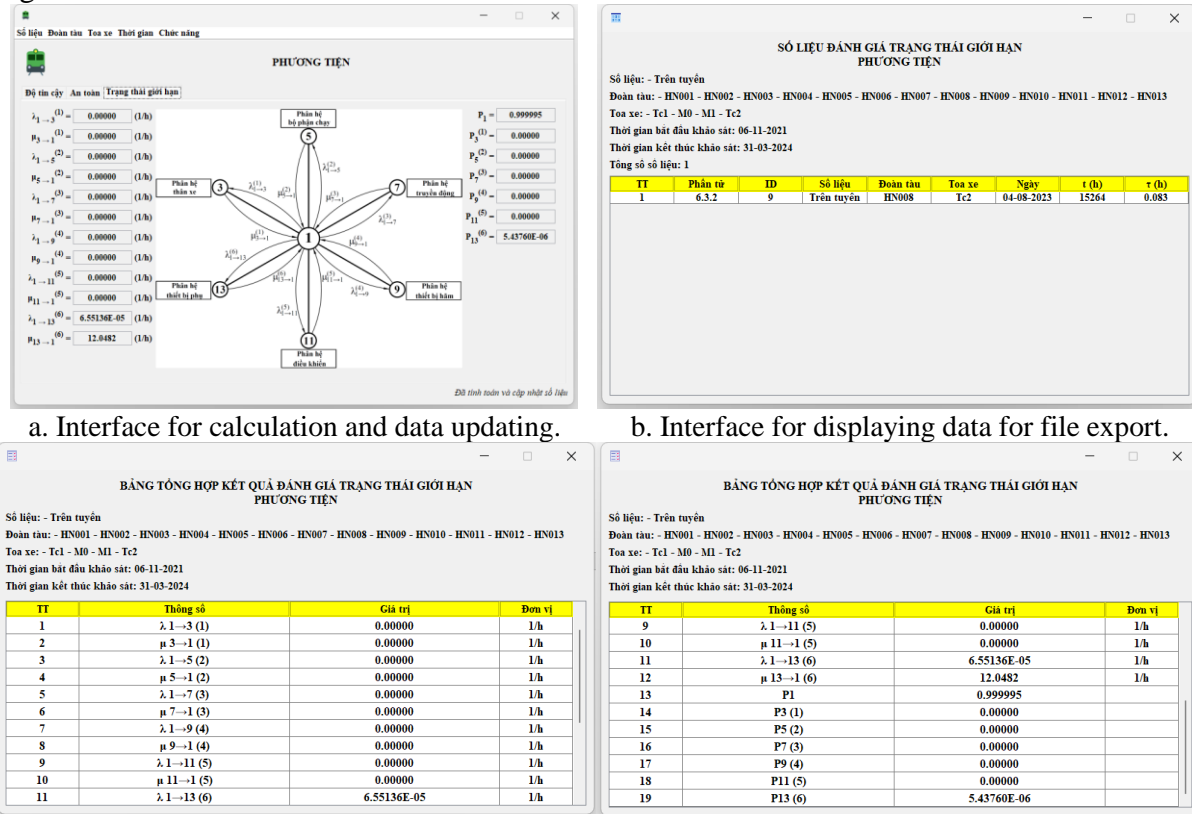


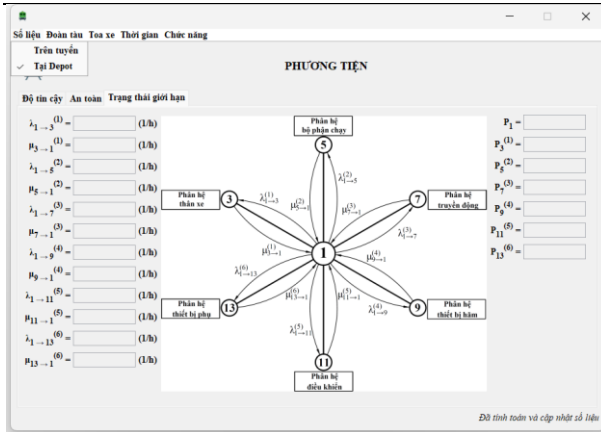
Figure 3. Interfaces for displaying evaluation results of the limiting probabilities when transitioning states of subsystems in the metro train during operation on the line.

The calculation results of the limiting probabilities when transitioning states of the subsystems and the rolling stock (metro trains) during operation on the line are presented in Table 3.

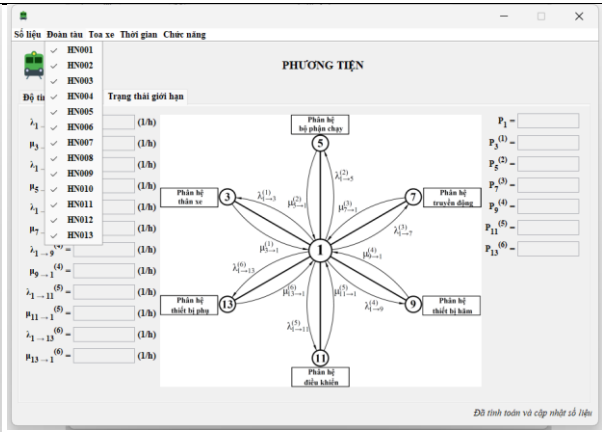
Thus, during operation on the line, the probability that the rolling stock remains in the operational state, or its reliability, is extremely high, with  $P_1 = 0.999995$ . The probability that the rolling stock is in a non-operational state due to a failure in the Auxiliary Equipment subsystem is very low,  $P_{13}^{(6)} = 5.4376 \times 10^{-6}$ . In other words, the reliability of the Auxiliary Equipment subsystem during the observed operational period is the lowest among all subsystems. However, its reliability value remains exceptionally high at 0.999995, which also represents the overall reliability of the rolling stock. The remaining subsystems are considered to have absolute reliability.

Table 3. Calculation results of the limiting probabilities when transitioning states of subsystems and rolling stock (metro trains) during operation on the line.

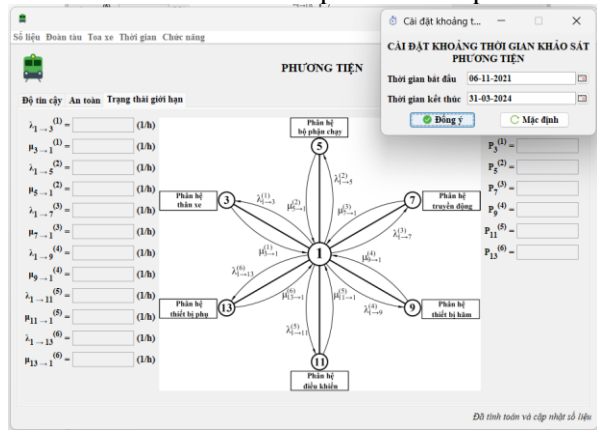
No.	Indicators	Limiting probability at the actual mean time between failures
1	Probability of the rolling stock being in an operational state $P_1$	0.999995
2	Probability of the rolling stock being in a non-operational state due to subsystem failures:	
2.1	Subsystem 1. Body $P_3^{(1)}$	0
2.2	Subsystem 2. Running Gear $P_5^{(2)}$	0
2.3	Subsystem 3. Transmission $P_7^{(3)}$	0
2.4	Subsystem 4. Braking Equipment $P_9^{(4)}$	0
2.5	Subsystem 5. Control $P_{11}^{(5)}$	0
2.6	Subsystem 6. Auxiliary Equipment $P_{13}^{(6)}$	$5.4376 \times 10^{-6}$



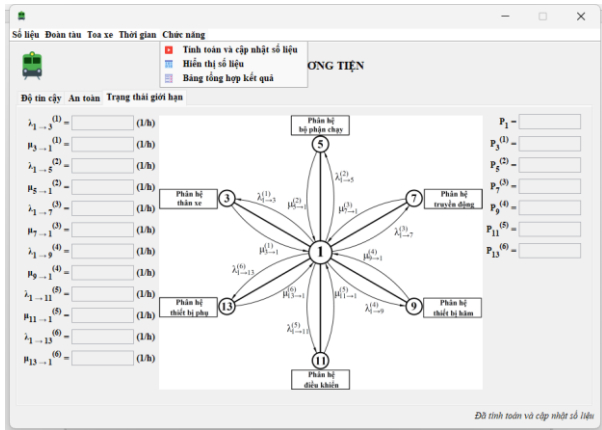
a. Interface for selecting statistical data during maintenance and repair at the depot.



b. Interface for selecting the number of metro trains for calculation.



c. Interface for selecting the survey period.



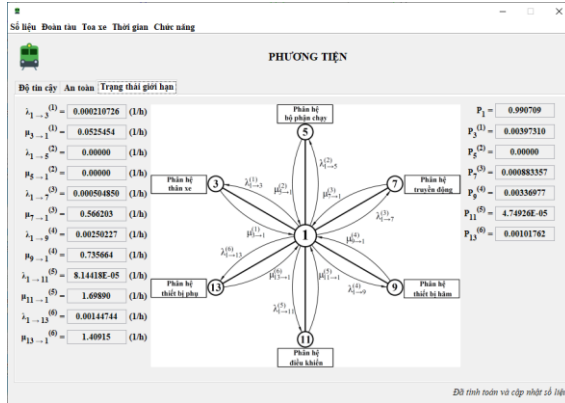
d. Interface for selecting calculation functions and displaying results.

Figure 4. Interfaces for selecting calculation functions of limiting probabilities when transitioning states for subsystems on metro trains during maintenance and repair at the depot.

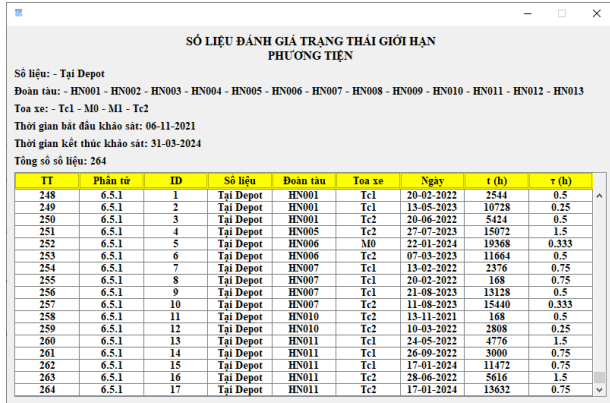


### 3.2. Evaluation of the limiting probabilities when transitioning states of subsystems on metro trains operating on the Cat Linh - Ha Dong line during maintenance and repair at the depot

The calculation process for evaluating the limiting probabilities when transitioning states of subsystems on metro trains operating on the Cat Linh - Ha Dong line during maintenance and repair at the depot is illustrated in Fig. 4 and 5.



a. Interface displaying the calculation results.



b. Interface displaying computed data for file export.



c. Interface displaying calculation results for file export.

Figure 5. Interface displaying the evaluation results of the limiting probabilities when transitioning states of subsystems on metro trains during maintenance and repair at the depot.

The calculation results determining the state transition limit probabilities of the subsystems and the rolling stock (metro trains) during operation on the line are presented in Table 4.

Thus, during maintenance and repair at the depot, the probability that the rolling stock remains in an operational state, or its reliability, is relatively high, with  $P_1 = 0.990709$ . The probability that the rolling stock is in a non-operational state due to subsystem failures, ranked from lowest to highest, is as follows: 1. Running Gear subsystem; 2. Control subsystem; 3. Transmission subsystem; 4. Braking Equipment subsystem; 5. Body subsystem. The Running Gear subsystem is considered to have absolute reliability.

However, to further enhance reliability and availability, it is necessary to increase the mean time between failures and reduce recovery time. Among these factors, reducing recovery time (improving the efficiency of maintenance and repair) is the most critical.

Table 4. Calculation results of the limiting probabilities when transitioning states of subsystems and of the rolling stock (metro trains) during maintenance and repair at the depot.

No.	Indicators	Limiting probability at the actual mean time between failures
1	Probability of the rolling stock being in an operational state $P_1$	0.990709
2	Probability of the rolling stock being in a non-operational state due to subsystem failures:	
2.1	Subsystem 1. Body $P_3^{(1)}$	0.00397310
2.2	Subsystem 2. Running Gear $P_5^{(2)}$	0
2.3	Subsystem 3. Transmission $P_7^{(3)}$	0.000883357
2.4	Subsystem 4. Braking Equipment $P_9^{(4)}$	0.00336977
2.5	Subsystem 5. Control $P_{11}^{(5)}$	$4.74926 \times 10^{-5}$
2.6	Subsystem 6. Auxiliary Equipment $P_{13}^{(6)}$	0.00101762

#### 4. CONCLUSION

The calculation results obtained over the survey period provide insight into the reliability of the subsystems of metro rolling stock on the Cat Linh - Ha Dong line during operation, as well as their availability and maintainability during maintenance and repair at the depot. Additionally, the most critical elements can be identified, enabling the operating unit to develop appropriate strategies for spare parts provisioning and replacement materials. This, in turn, helps reduce downtime for maintenance and repair, thereby improving reliability and availability.

The above calculations correspond only to the operational period from November 6, 2021, to March 31, 2024. Moving forward, it is necessary to continuously monitor, collect, and update statistical data on failures causing service disruptions and unscheduled repairs at the depot for individual elements. This will allow for periodic calculations over different time intervals, such as annually or over multiple years, serving as a basis for analyzing, comparing, and evaluating reliability indicators for specific operational phases and the entire service life of the rolling stock.

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