Transport and Communications Science Journal

DETERMINE THE INFLUENCE FACTOR OF GEOMETRICAL PARAMETERS ON THE DYNAMIC CHARACTERISTICS OF THE MICRO CRAB-SHAPED BEAM

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ARTICLE INFO

TYPE: Research Article
Received: 31/03/2025
Revised: 24/04/2025
Accepted: 10/05/2025
Published online: 15/05/2025
https://doi.org/10.47869/tcsj.76.4.7

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Abstract. Microelectromechanical devices are being used much more in science and technology to reduce energy consumption and increase accuracy. The design of mechanical structures in these devices requires reducing the cost of testing and prototype manufacturing. It is necessary to define the essential parameters in designing the microstructures to reduce the design time. This paper presents a simulation method to determine the influence of the geometrical parameters of the micro-beam models with Crab-shaped used in the micro-mechanical structures on the equivalent stiffness. The significant parameters influencing the rapid change of equivalent stiffness are shown using the local sensitivity analysis technique and the surface response in the ANSYS Workbench. The results show that the length of the sensing bar with the convex curve and the maximum deviation is 76% for the equivalent stiffness, while the width of the beam with the concave curve gives the maximum value of 400% at the same 70% of the variation of each geometrical parameter. The local sensitivity analysis values for the length of the driving and sensing bar, and the width of the crab-shaped beam are -1.4485, -0.2786, and +2.0355, respectively. These analysis results are essential for choosing the suitable value for the geometrical parameters of the micromechanical structure according to the proposed resonant frequency in further studies.

Keywords: FEM analysis; micro-beam; equivalent stiffness; Crab-shaped beam.

1. INTRODUCTION

Every movement of the elements in the micro-vibrating system has a significant impact on the overall functionality of the system. In particular, in Micro-Electro-Mechanical System (MEMS) devices, the role of micro beams is crucial as they transmit motion from one component to another while directing the movement of the mass element. In fact, various types of mechanical micro-beams are designed and utilized. Nowadays, the most common ones are such as the single model [1-3], Crab-shaped model [3-4], multi-folder model [4-5], V-shaped model [6], surpentine shape [7], and coupled elastic beam [8-9]. These beams all have one fixed ending and the other ending suspends a mass element allowing it to move in the purposed directions to carry out the necessary vibration to the operation principle of the mems device that contains these micro beam structures. The single beam in [2] was known as a Timoshenko beam with micrometer scale. This is the simplest type of microbeam in MEMS. Some typical applications of these beams are known as in the models of MEMS vibratory gyroscopes [4], or one-axial, 2-axial, and 3-axial accelerometers [2], [11]. They are considered resonant systems and operate at their resonant frequency bandwidth. Recently, the analysis of small-scale structures at the micro- and nano-levels has attracted considerable interest due to its intriguing aspects, the readers may refer to the works [12-13] for more detailed information.

In the mechanical structure of MEMS devices, the flexible beams are used to guarantee the motion of the moveable parts. The geometrical parameters of these beams (length, width, thickness, number of bars, etc..) decisively affect their equivalent stiffness and thus affect the resonant frequencies of the micro-resonators in MEMS devices [6]. During the design of micro-mechanical systems, it is very important to choose a set of dimensions that match the design intent. Some structural parameters play essential roles and determine significantly the variation of the dynamic characteristics of the oscillation system. Getting the desired set of parameters requires testing, and knowing which parameters play an important role in deciding the design object will significantly reduce the time of the design process.

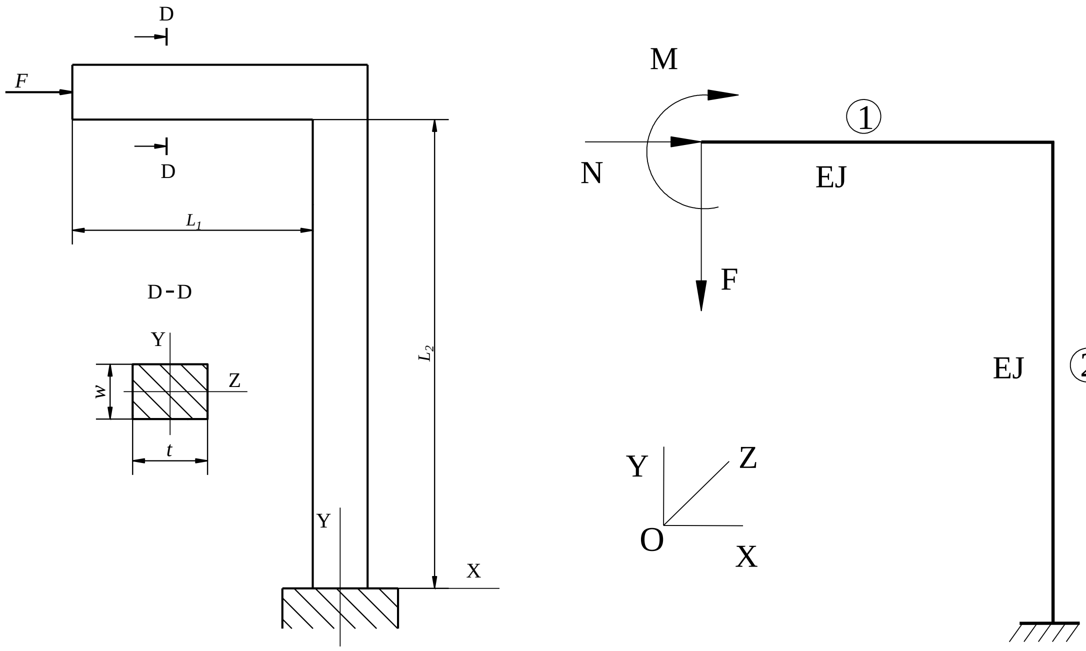
This paper focuses on determining the important dimension parameters that affect the operating frequency through the effective mass and the equivalent stiffness coefficient for the common type of regular beam with a Crab shape used in MEMS device structures. The finite element method and simulation method with the ANSYS Workbench software are used in this work to obtain the dynamic characteristics of the mentioned microbeam. This database is necessary to orient the design process of MEMS devices.

2. CONFIGURATION OF MICRO CRAB-SHAPED BEAM

The Crab shape beam is used in the MEMS vibratory gyroscope (MVG) model [4]. The configuration of the Crab-shaped beam is presented in Fig. 1, situated inside the Cartesian *XYZ* coordinate system. Each beam consists of two perpendicular bars with suitable dimensions to allow the proof mass to oscillate in the corresponding directions. The length and width of the bars are L and w, respectively. The thickness of beams is t, so the cross-section in a rigid body is $w \times t$. Generally, the bar with L_1 length is the driving beam and the bar with L_2 length is the sensing beam. They allow the proof mass of the MVG model to move in two perpendicular directions. In order to build 3D models conveniently, the initial parameters are chosen as follows: L_1 = 100 µm; L_2 = 300 µm; w = 10 µm; t = 30 µm. The boundary conditions are defined in that one ending beam of the beam is fixed, while the other end suspends a mass element allowing it to move in the purposed direction (*y* direction) as shown in Fig. 1a.

The material used commonly for this beam is the Silicon in SOI wafers. The material characteristics of Silicon are shown in Table 1.

The computational model of the crab-shaped beam is built with the active forces as shown in Fig. 1b, where: *M* represents the reaction moment connected in the *OXY* plane, and *N* represents the reaction force with a direction along the *OX* axis. The active force *F* is in the *y*-direction. The material and cross-section characteristics are represented by *EJ*.



a) Configuration of the micro crab-shaped beam b) Diagram of active forces on the crab beam

Figure 1. A common type of crab-shaped beam.

Table 1. The properties of the Silicon material.

Property	Value	Unit
Density	2330	kg/m^3
Poisson's ratio	0.28	-
Young's modulus	1.69×10^{11}	Pa
Bulk modulus	1.2879×10^{11}	Pa

3. DYNAMIC CHARACTERISTICS OF THE MICRO CRAB-SHAPED BEAM

The eigenfrequency or natural frequency of the mechanical system plays a critical role in resonance phenomena. When an external force is applied to a mechanical system at or near its natural frequency, the system can respond with large-amplitude vibrations. It allows the determination of the necessary driving force applied to a MEMS component to obtain the large displacements to the desired operating range. Moreover, determining the natural frequency of oscillation can help us design structures with the stiffness of the beams, thereby yielding the operating frequencies for each mode of oscillation to avoid coupling between different modes of oscillation.

The governing differential equation for the resonant system is as follows:

$$[M]\ddot{\vec{q}} + [K]\vec{q} = \vec{0} \quad (1)$$

where, [*M*], [*K*], and $\vec{q}$ are the inertia matrix, stiffness matrix, and displacement of the vibration system.

Commonly, the natural frequency of the simple structure can be determined as follows:

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} \quad (2)$$

where: *K* is the equivalent stiffness of the beam and *M* is the effective mass of the entire structure due to the displacement of the mass element.

In this work, the dynamic characteristics of the micro Crab-shaped beam are first considered to determine. They consist of two main factors: the effective mass and the equivalent stiffness.

3.1. Effective mass

Besides, the effective mass of the beam also needs to be determined to calculate the natural frequency in expression (1). For the convenience of calculation, it is assumed that the velocities of the sectional elements of the beam are proportional to their displacements and the mass distribution is uniform along the beam length. In this case, the effective mass of the beam is determined based on the principle that the kinetic energy of the effective mass is equal to the sum of the kinetic energies of all moving parts on the beam [10]. Fig. 2 illustrates the general schematic for calculating the effective mass of the beam.

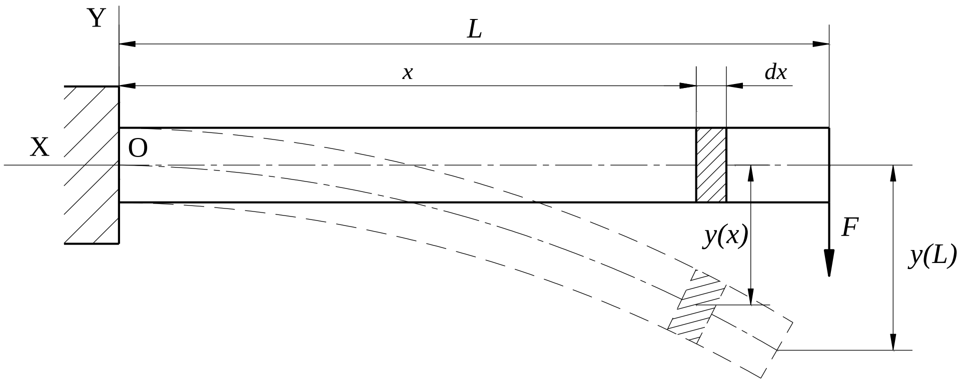


Figure 2. General diagram of beam displacement.

The effective mass of the mass elements can be determined as follows [4], [10]:

$$m_{eff} = \sum_{i=1}^{n} \frac{m_i}{L_i} \times \int_0^{L_i} \left(\frac{\dot{y}_i(x)}{\dot{y}_i(L_i)} \right)^2 dx \quad (3)$$

According to the assumption, we have:

$$\frac{\dot{y}_i(x)}{\dot{y}_i(L_i)} = \frac{y_i(x)}{y_i(L_i)} \tag{4}$$

Consider the proposed beam in Fig. 1, the structure of the micro crab-shaped beam consists of two segments numbered 1 and 2. The effective mass of the beam will be equal to the sum of the effective mass of each segment, relative to the starting position of that section. In the case of the crab beam with m_C weight and one fixed ending, the vibration point set at the other ending, the effective mass converted at the end of the crab beam is defined as follows:

$$m_{eff} = \frac{33}{140}.m_C \tag{5}$$

The expression (5) only reflects the effective mass value of the crab beam when considered by itself. The effective mass value of the oscillating system also depends on the mass of the inertial element when the crab beam is connected with some other elements in the system.

3.2. Equivalent stiffness

The equivalent stiffness of the beam needs to be determined to receive the natural frequency of the beam or the oscillation system. Accordingly, the equivalent stiffness of a crab beam can be determined analytically based on the energy method [10]. For the beam structure in Fig. 1, the equivalent stiffness of the crab beam in the *x* and *y*-direction can be determined using the formula [10-11], [14]:

$$K_X = \frac{Et}{4}\left(\frac{w}{L_2}\right)^3\left(\frac{L_1 + 4L_2}{L_1 + L_2}\right);\ K_Y = \frac{Et}{4}\left(\frac{w}{L_2}\right)^3\left(\frac{4L_1 + L_2}{L_1 + L_2}\right) \tag{6}$$

Considering the assumption that during the working process, the crab beam deforms within the elastic limit of the material. The equivalent stiffness of micro-beams is defined as the ratio of the acting force F and the displacement δ of the structural beam at the point where the force applies:

$$K = \frac{F}{\delta} \tag{7}$$

where the displacement δ is determined by using the simulation method in ANSYS Workbench.

The previous research showed that the stiffness of the beam determined by the simulation method achieves a more correct result than using the analytical method, thanks to taking into account the individual mass of the beam. In this work, ANSYS Workbench was used to simulate and determine the equivalent stiffness of the crab beam with the variation of the geometrical parameters. The thickness of the whole beam is fixed at 30 µm. The length of the driving bar is in the range of 300 ÷ 500 µm with a rise of 66%. The length of the sensing bar is in the range of 100 ÷ 300 µm, and 200% rise in the length. The last parameter is the width of the whole beam *w*. This factor is changed by 120%, rising in the range of 10 ÷ 22 µm.

Fig. 3 and Fig. 4 represent the response of the equivalent stiffness with a variation of the length beam in one direction (*x*-direction) in both of the methods mentioned above with the red dashed line for the results of the analytical method and the blue solid line for the simulation method. Fig. 3 shows that the length of the driving causes a slight change in the equivalent stiffness K_x of the crab beam (from 67 N/m at 100 µm down to 54 N/m at 300 µm) when the width is small (w=10 µm). The deviation rises with the rise of the width in both cases of changing the length of the two bars. The change in stiffness K_x becomes more obvious when the width of the beam increases and the length of the sensing bars changes at the same time (see Fig. 4). The change level is up to 75% (from 1399 N/m at 300 µm length of the sensing bar down to 339 N/m at 500 µm). Fig. 5 expresses the effect of the width beam on the equivalent stiffness K_x. The results show that this parameter strongly affects the stiffness K_x. In the 500µm length of the sensing beam, the increase of 120% of the width beam causes up to 9 times the rise of the K_x.

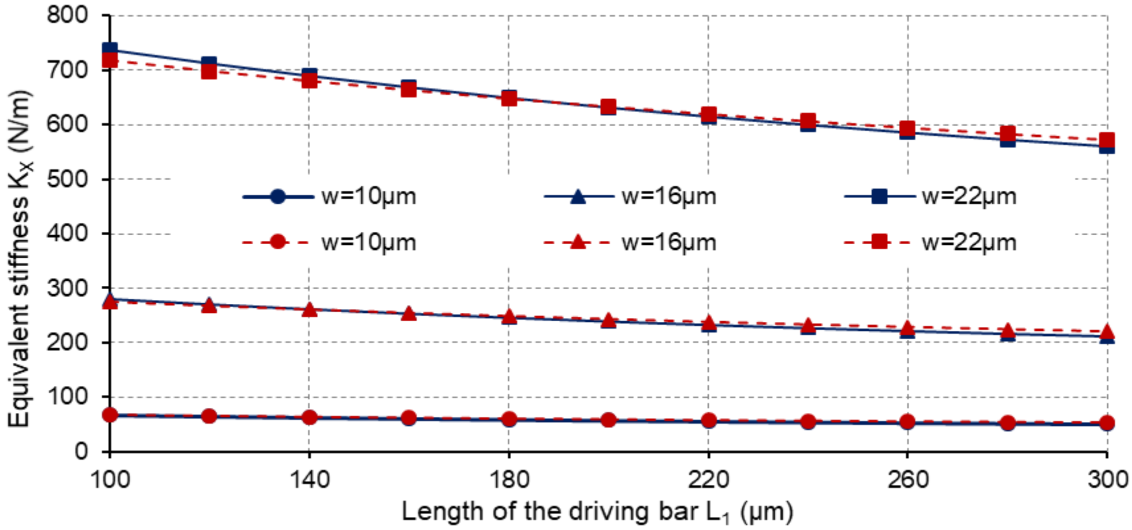


Figure 3. Variation of the equivalent stiffness to the length of the driving bar.

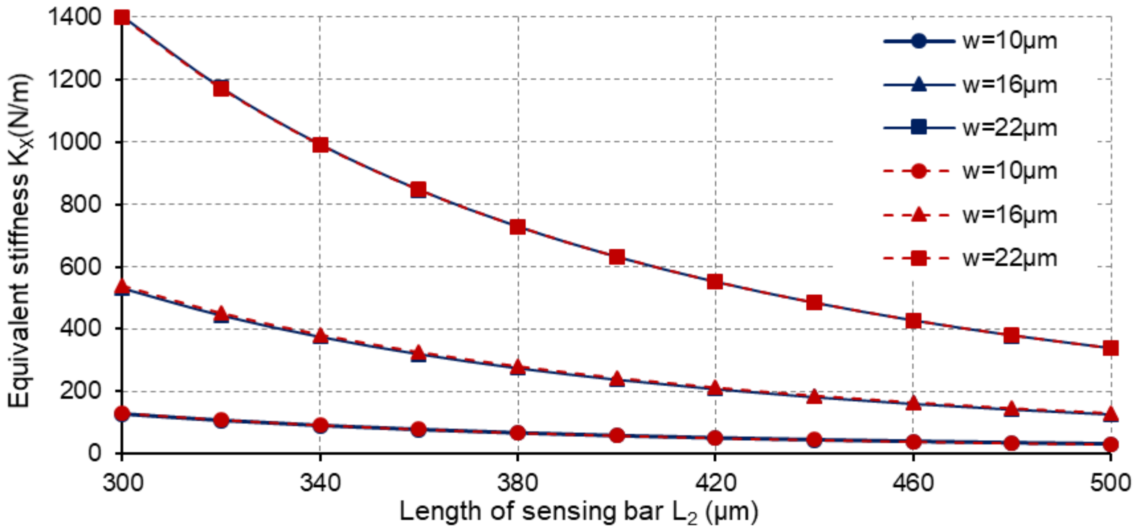


Figure 4. Variation of the equivalent stiffness to the length of the sensing bar.

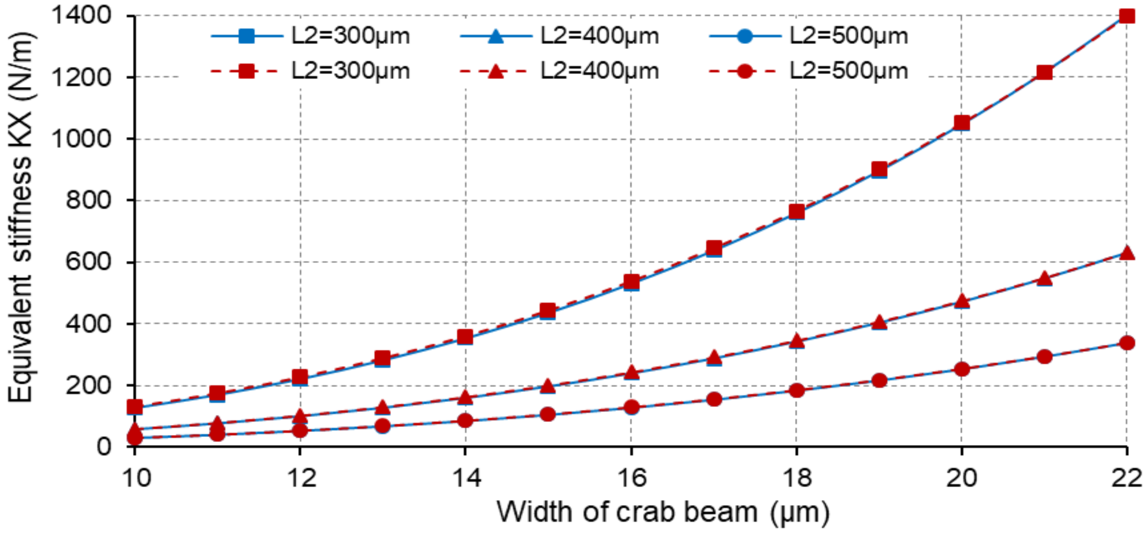


Figure 5. Variation of the equivalent stiffness to the width of the beam.

The influence of both lengths of bars on the equivalent stiffness K_x can be observed in Fig. 6 with the surface response built by data obtained from simulation in ANSYS. With the same technique, the surface responses describing the relation between the equivalent stiffness K_x and the three mentioned dimensions of the micro crab-shaped beam are displayed in Fig. 7 and 8. The equivalent stiffness K_y can be obtained by using the same method and is not mentioned in this work.

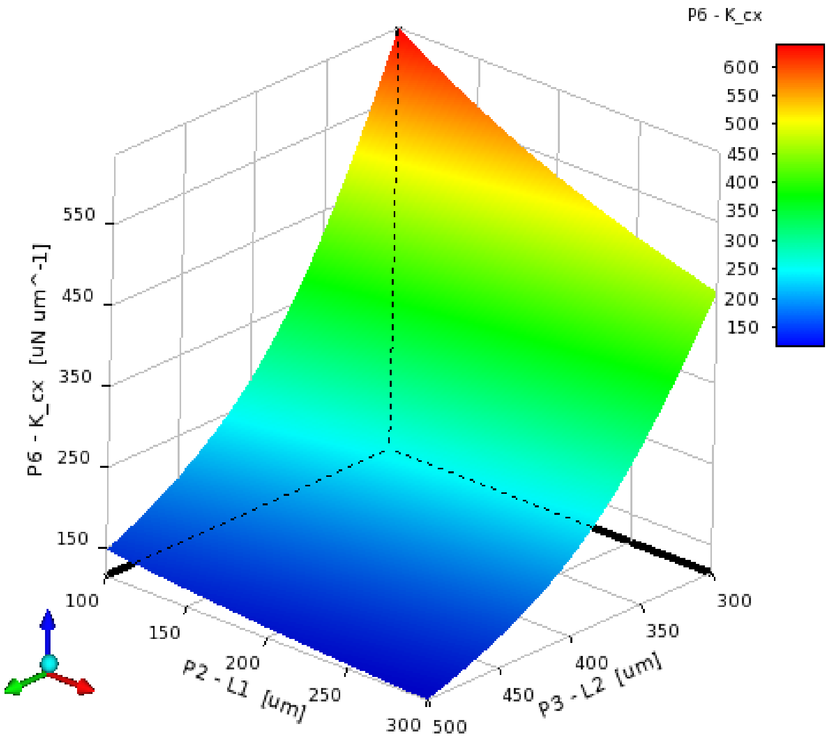


Figure 6. Surface response of K_x versus the lengths of two bars.

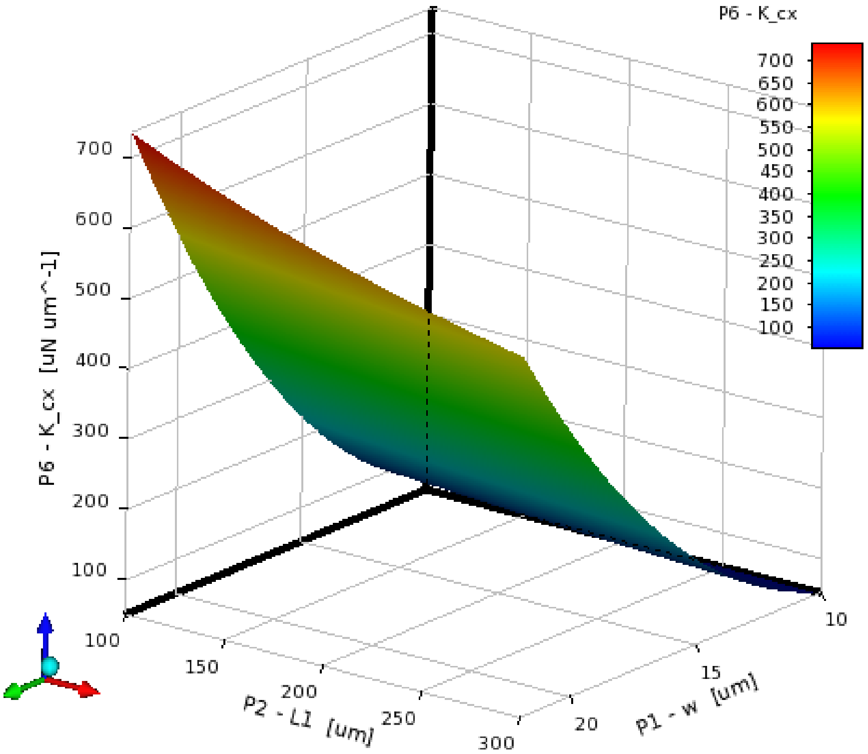


Figure 7. Surface response of K_x versus the length of the driving bar and width beam.

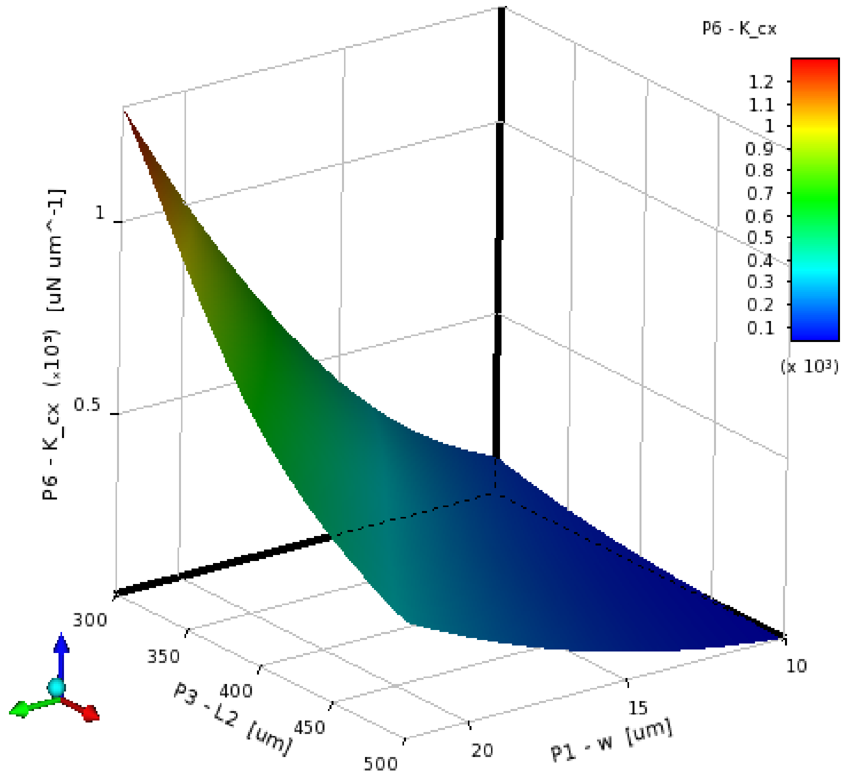


Figure 8. Surface response of K_x versus the length of the sensing bar and width beam.

4. IMPACT OF THE GEOMETRICAL PARAMETERS ON THE EQUIVALENT STIFFNESS OF THE CRAB BEAM

In order to evaluate the effect of the main geometrical parameters on the equivalent stiffness of the crab beam, the data about the equivalent stiffness are collected when all three geometrical parameters of the crab beam vary in the surveyed range. Besides, the local sensitivity analysis (LSA) technique is also applied. Local sensitivity analysis is the assessment of the local impact of input factors' variation on the model response by concentrating on the sensitivity in the vicinity of a set of factor values. Such sensitivity is often evaluated through gradients or partial derivatives of the output functions at these factor values, i.e. The values of other input factors are kept constant when studying the local sensitivity of an input factor. Local sensitivity can support determining the extent to which each input parameter influences the output parameter.

In this work, the central composite design (CCD) in the design of experiment type in ANSYS software was used to create the data for the LSA with the input parameters including the length beam and the width beam for the crab beam.

The results of LSA for this beam are also shown in Fig. 9 with three values for each respective parameter. The LSAs for the driving bar, sensing bar, and width of the beam are -1.4485, -0.2786, and +2.0355, respectively. The negative values of LSA results indicate that the corresponding parameters have an inverse effect on the output data. In contrast, the positive ones create a proportional influence on the output factor of the system. The results describe that the width of the beam affects the equivalent stiffness of the beam more strongly than its length. The positive value of LSA for the width beam means this geometrical parameter is directly proportional to the equivalent stiffness of the beam. On the opposite side, the negative values LSA for two lengths of bars describe that their effect is inversely proportional to this stiffness.

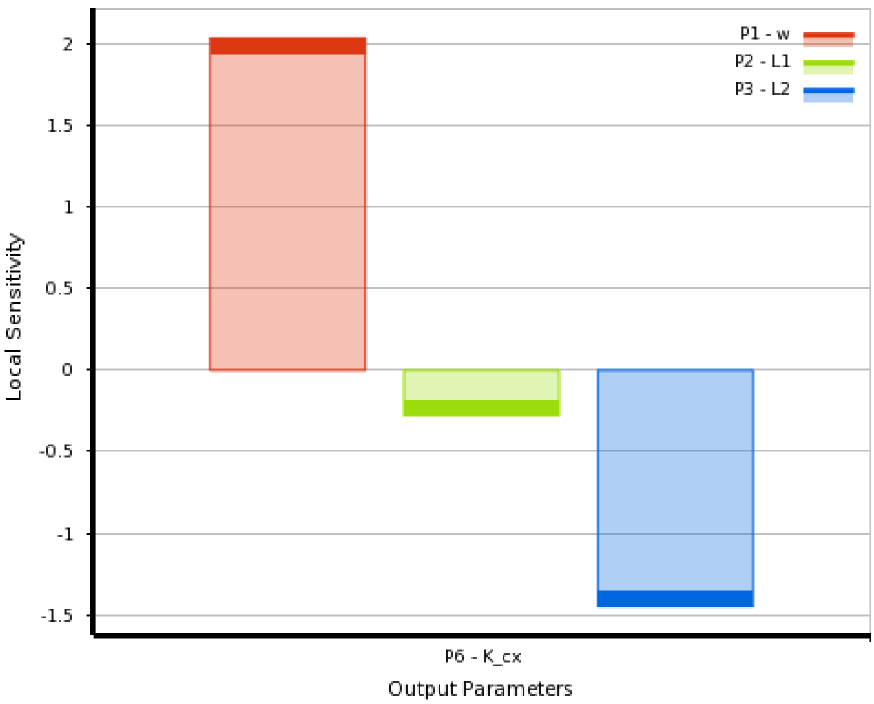


Figure 9. LSA for the proposed crab beam.

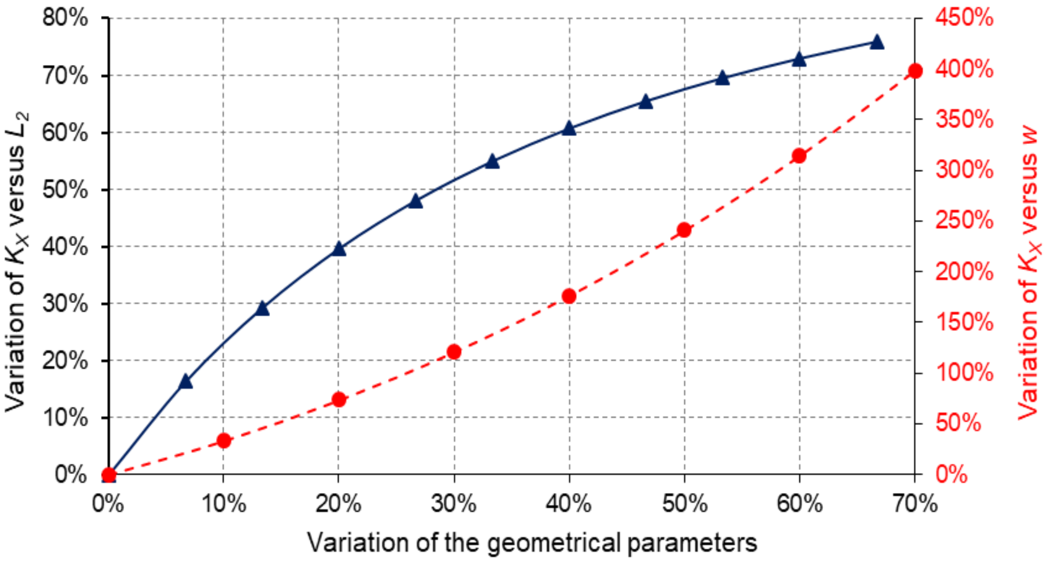


Figure 10. Variation of K_x versus the length of the sensing bar and width beam.

The variation of the equivalent stiffness in the *x*-direction K_x to the variation of the length of the sensing bar and the width of the beam is depicted in Fig. 10 (the blue solid curve for the variation of the stiffness versus length of bar, and the red dashed curve for the width of the beam). It can be seen that the length of the sensing bar with the convex curve and the maximum deviation is 76% for the equivalent stiffness, while the width of the beam with the concave curve gives the maximum value of 400% at the same 70% of the variation of each geometrical parameter. This result supports that in the designing process, it is necessary to attend to the width parameter of this beam to preliminarily adjust the equivalent stiffness of the system change the length of the beam to fine adjustment, and obtain the desired output parameter for the micro-mechanical structure.

5. CONCLUSIONS

The paper presents a method for determining characteristic dynamic quantities for a type of microbeam with complex structure, analyzing the influence of geometric parameters such as length and width of the beam structure on the effective mass and equivalent stiffness of the crab beam commonly used in MEMS devices. Using the analysis tool of ANSYS software, important and most influential dimensions for the equivalent stiffness of the beams have been identified. The width of the beam has the greatest influence on the output parameter, with an increase of 70% in the width and sensing bar of the beam, resulting in a respective decrease in K_x stiffness by 400% and 76%. Both lengths in the driving and sensing bar of the beam have less impact on the stiffness. The local sensitivity coefficients for these lengths are -1.4485 and -0.2786, respectively, while the local sensitivity coefficient for the width beam is +2.0355. The results of the study provide a basis for analyzing more complex structures and serve as a foundation for optimizing the structure to reduce survey time and computational requirements for mechanical oscillation structures in MEMS devices.

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