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PERFORMANCE EVALUATION OF THE ARTIFICIAL HUMMINGBIRD ALGORITHM IN THE PROBLEM OF STRUCTURAL DAMAGE IDENTIFICATION

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Abstract. Recently, Structural Health Monitoring (SHM) has become a critical component of the maintenance and safety of lifeline infrastructures such as dams, skyscrapers, and bridges, thanks to its ability to detect structural failures at the early stages. In this paper, we evaluate the performance of the SHM damage identification tool using a novel metaheuristic algorithm called the Artificial Hummingbird Algorithm (AHA). The proposed approach is evaluated by two case studies of different bridge structures in Vietnam with different simulated damage scenarios. The potency of the AHA is compared against the other wellknown metaheuristic algorithms such as Cuckoo Search (CS), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Teaching-Learning Based Optimization (TLBO). The results show that the AHA performs much better than the other algorithms in terms of accuracy and computational cost. The application of AHA can help to reduce the cost and time required for structural maintenance significantly, as well as improve the lifecycle of the structure.

Keywords: damage identification, Artificial Hummingbird Algorithm, metaheuristic algorithms, damaged bridge structure.

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1. INTRODUCTION

Structural health monitoring (SHM) is a vital task in ensuring the safety and longevity of civil infrastructures, such as bridges, buildings, and dams. SHM involves monitoring the structural integrity of these infrastructures using various sensors and techniques to identify potential damages, such as cracks, corrosion, or deformation. The early detection of such damages can allow for timely repairs, reducing the likelihood of catastrophic failure and minimizing maintenance costs. Recently, to improve the efficacy of damage identification features of SHM, different metaheuristic optimization algorithms have been successfully applied [1–5]. Those algorithms are particularly useful in SHM because they can handle the large and complex data sets produced by the sensors used in the SHM and can quickly identify the optimal solution to the problem.

One of the most significant benefits of metaheuristic algorithms is that they do not call for any prior understanding or its constraints. This makes them highly adaptable and applicable to different optimization problems. Moreover, metaheuristic algorithms can efficiently search through large and complex search spaces, which can be difficult or impossible to solve using traditional optimization techniques.

Figure 1. Categories of metaheuristic algorithms.

There are various ways to classify metaheuristic algorithms, but one possible way is to categorize them based on the inspiration source for their design (Figure 1). Among them, evolutionary-based and nature-based algorithms are proven to be the most effective due to their ability to obtain the most accurate solutions to complex problems, which can be impossible for traditional optimization algorithms to deal with:

Evolutionary-based algorithms are inspired by the process of natural selection and evolution. These algorithms simulate the process of natural selection, where the fittest individuals are chosen for reproduction and create offspring that inherit their favorable qualities by using iterative, stochastic approaches to seek the best solutions to a problem. Some evolutionary-based algorithms such as Genetic Algorithm (GA), Evolutionary Algorithm (EA), Differential Evaluation (DE), Clonal Selection Algorithm (CSA),…Hao et al. [6] used vibration-based methods for detecting structural damage. By minimizing the objective function that compares changes in measurements before and after damage, a genetic algorithm with real

number encoding is utilized to detect structural damage. The proposed method is validated on a cantilever beam and a frame, and results show that even in cases where the analytical model is inaccurate, genetic algorithms can identify damaged components correctly.

Nature-based metaheuristic algorithms are inspired by ecological and biological systems. These algorithms simulate the behavior of natural systems and use their principles to solve optimization problems. Besides, nature-inspired algorithms can also be divided into subcategories such as Bio-inspired, Swarm-based, Plant-based, Human-based, and Physics/Chemistry-based.

Bio-based metaheuristic algorithms mimic the behaviors of natural organisms and their evolution to find optimal solutions to complex problems. Bio-based metaheuristic algorithms are stochastic and have the ability to explore a large search space for optimal solutions. Tran et al. [7] presented a method of damage detection in bridges and beam structures by improving Artificial Neural Network (ANN) training parameters using Cuckoo Search (CS). The proposed te ANN-CS is more precise in identifying and measuring structural damage and also requires less computational time.

Swarm-based metaheuristic algorithms are inspired by the animal's social behaviors in groups such as flocks of birds, swarms of bees, or schools of fish. Examples include Particle Swarm Optimization (PSO), Bee Algorithm (BA), Bacterial Foraging Optimization (BFO),…Wei et al. [8] show that the effectiveness and resilience of the adapted PSO algorithm were confirmed using three different civil engineering structures. The outcomes indicate that the technique is highly effective and successful in identifying structural damage, even when Gaussian noise is present.

Plant-based metaheuristic algorithms simulate the behaviors of plants and their ability to adapt to changing environmental conditions to find optimal solutions to complex problems. Examples include Invasive Weed Optimization (IWO), Artificial Root Foraging Algorithm (ARFA), Flower Pollination Algorithm (FPA), Phototropic Optimization Algorithm (POA), Sunflower Optimization (SFO),…Gomes et al. [9] proposed a sunflower optimization algorithm (SFO) technique for multi-modal problems. The proposed SFO provides resilience compared to conventional algorithms by using concepts like root velocity and pollination. The inverse problem of structural damage detection in composite laminated plates is then tackled using the new approach with a high level of accuracy.

Human-based metaheuristic algorithms are designed to mimic the way humans solve complex problems, such as decision-making, optimization, and search problems. Examples include Seeker Optimization Algorithm (SOA), Imperialistic Competitive Algorithm (ICA), Biogeography Based Optimization (BBO), Teaching-Learning-Based Optimization (TLBO),…Behrouz et al. [10] proposed a new method that was utilized to detect structural damage by applying a modified teaching-learning optimization algorithm. This approach involved using a unique damage factor that considered modal flexibility and strain energy while accounting for environmental changes. The results indicate that the modified teaching–learning optimization algorithm (MTLBO) approach had a faster convergence rate for the objective function and provided more accurate estimates of the extent of the damage, even in the presence of Gaussian noise.

Physics/Chemistry-based metaheuristic algorithms are motivated by physical and chemical processes, such as the laws of thermodynamics, the laws of gravity, or the laws of motion. Examples include Gravitational Search Algorithm (GSA), Vortex Search Algorithm (VSA),

Lightning Search Algorithm (LSA),…A novel metaheuristic algorithm named ASCA-DE (Adaptive Sine Cosine Algorithm integrated with Differential Evolution) was proposed by Bureerat [11] and was developed to identify defects in the tested structures. The findings indicate that ASCA-DE surpasses several well-established metaheuristic algorithms in terms of performance.

Overall, each type of metaheuristic algorithm has its own strengths and limitations, depending on the specific problem at hand. Therefore, it is essential to carefully choose and apply the most appropriate algorithm to solve each optimization problem effectively. Among the latest nature-based algorithms, the Artificial Hummingbird Algorithm (AHA) has quickly gained its status as one of the superior methods for solving optimization problems [12–15]. Inspired by those unique traits of the hummingbird, the Artificial Hummingbird Algorithm (AHA) was introduced in 2022 by Zhao et al. [16]. The AHA algorithm imitates hummingbirds' distinctive flight characteristics and smart foraging strategies in the wild. In this research, we will evaluate the performance of the AHA algorithm in detecting damages in different bridge structures with different simulated damage scenarios. The efficiency of the studied AHA is compared against the other well-known metaheuristic algorithms such as Cuckoo Search (CS), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Teaching-Learning Based Optimization (TLBO). The results are then compared and discussed.

2. THE ARTIFICIAL HUMMINGBIRD ALGORITHM

Usually considered the smallest kind of bird ever existed on Earth, the hummingbird is also one of the most intelligent animals. For the body-to-brain ratio, their intelligence surpasses that of humans [17]. Moreover, although small in size, the hummingbird has an incredibly flexible flying skill. A hummingbird is able to change direction while flying at different altitudes with high precision. Their flight versatility helps them not only to fly durably but also to navigate effectively during food foraging. The algorithm remodels three different foraging strategies that hummingbirds have used to ensure their sustained access to food sources: guided foraging, territorial foraging, and migrating. Altogether they generate a robust natural-inspired optimization algorithm that can help researchers to solve different mathematical and engineering problems.

3.1.Initialization

Given *n* population size of hummingbirds, for *n* number of food sources can be initialized as Eq. (1) below:

$$
x_i = LowBoundary + r \cdot (UpBoundary - LowBoundary) \quad i = 1, ..., n \tag{1}
$$

Where LowBoundary and UpBoundary represent the Lower and Upper Boundaries of the search, r is a random parameter of [0, 1], x_i indicates the location of the ith food source.

Another component will be initialized in the initialization process, which is the Visiting Table (VT). The VT indicates the information that the hummingbird can store about a specific food source, such as the duration since its last visit to a food source. The likelihood that the hummingbird would find more food there increased with the length of time since its last visit. It is logical that the bird would have the tendency to visit these food sources first. The visiting table is initialized as:

$$
VisitingTable_{i,j} = \begin{cases} 0 & \text{if } i \neq j \\ \text{null} & i = j \end{cases} i = 1, ..., n; j = 1, ..., n \tag{2}
$$

When $i \neq j$, Visiting Table_{i,j} = 0, which indicates the ith humming bird in the current iteration has been to the jth food source. When $i = j$, the hummingbird is foraging food at its specific food source.

3.2.Food Foraging

The hummingbird conducts three foraging strategies to search for food: guided foraging, territorial foraging, and migration foraging. All three tactics are used in order to guarantee that the hummingbird will have access to the best-surrounding food supply.

Guided foraging

The guided foraging strategy of AHA is inspired by the disposition of the hummingbird to look for the food supply with the highest capacity possible. This can be achieved by simply searching for the food source with the longest unvisited time from the visiting table. Once the target is identified, the hummingbird shall fly towards it for food. The guided foraging of hummingbirds can be expressed as:

$$
v_i(t+1) = x_{i,tar}(t) + a \cdot D \cdot (x_i(t) - x_{i,tar}(t))
$$
\n(3)

Where $v_i(t)$ is the updated position after the foraging, $x_i(t)$ is the position of the i^{th} food source at time t, $x_{i, tar}(t)$ is the position of the targeted food source that the ith hummingbird tends to fly to, $a \sim N(0,1)$ is a guiding coefficient, and D is the coefficient for the flight path of the hummingbird.

Territorial foraging

In territorial foraging, a hummingbird will typically fly to an area with a novel food source rather than return to the older ones after consuming the food source. A hummingbird will move to a nearby location of its territory so that it can look for a different potential solution from the existing one. The territorial foraging tactic is given as given in Eq. [\(4\)](#page-4-0):

$$
v_i(t+1) = x_i(t) + b \cdot D \cdot x_i(t) \tag{4}
$$

Where $b \sim N(0,1)$ is a territorial factor, and D is the coefficient for the flight path of the hummingbird.

Migration foraging

When a hummingbird visits a region too regularly, the food source within the area begins to get depleted. When that happens, the hummingbird must migrate to a different region to look for food. The hummingbird randomly selects a random food source to fly to, and the visiting table will be updated once the process is completed. The migration from the nearly depleted food source to a new random food source can be indicated by the following equation:

$$
x_{depleted}(t+1) = LB + r \cdot (UB - LB)
$$
\n⁽⁵⁾

Where $x_{depleted}$ is the location where the food supply is almost depleted

Figure 2. Three foraging behaviours of AHA.

Inputs:	
Population size n , Perform equation (1)	
Number of iterations n_{iter}	
Position of hummingbird x_i ,	
Lower Boundaries, Upper Boundaries	
Perform equation $(2(1)$	
Initialization	
While $t < n_{iter}$ Do	
For i^{th} population from 1 to <i>n</i> Do	
If rand < 0.5	
Then	% Guided foraging
Perform equation (3)	
Update visiting table	
Else	% Territorial foraging
Perform equation (4)	
Update visiting table	
End If	
End For	
If $mod(t,2n) = 0$ Then	% Migration foraging
Perform equation (5)	
Update visiting table	
End If	
End While	
Return best Fitness values and x_i	

Figure 3. Pseudo-code of AHA.

3. CASE STUDIES FOR STRUCTURAL DAMAGE IDENTIFICATION

3.1. Ba Thap bridge

In the first case, a simple girder bridge is used as the case study application. Ba Thap Bridge ([Figure 4](#page-6-0)) is located at Km1541+831, QL1, in Ninh Thuan province and is chosen for the validation of the algorithm. The width of the bridge is 10.5m with two lanes. The bridge consists of two simple inverse T-beam spans of 15m each. The bridge piers are made of reinforced concrete. Some other structures include rubber bearings, single rail expansion joints, and

reinforced concrete balustrades with steel handrails. This type of structure is also typical in Vietnam and is used for flatland and midland regions with the ability to cross small rivers and streams.

Figure 4. Ba Thap bridge and its cross-section of the inverse T-beam.

Finite element model

To analyse the dynamic characteristics of the structure, a Finite Element model (FE) of the bridge is constructed using the Stabil toolbox [18] of the MATLAB program. The model is constructed using 30 nodes and 29 elements accordingly [\(Figure 5\)](#page-6-1), with the boundary conditions of one fixed bearing and one movable bearing at the two ends of the bridge. Some parameters of the bridge are presented in Table 1. below:

Parameter		Symbol	Value	Unit	
Geometric properties	Cross-section area	A	0.2655	$\rm \dot{m}^{2}$	
	Moment of Inertia Ixx	Ixx	8.2023×10^{-3}	$\rm \dot m^4$	
	Moment of Inertia Iyy	Iyy	1.1182×10^{-2}	m^4	
Material properties	Young's modulus	Е	3.06×10^{7}	(kN/m ²)	
	Density		2450	(kg/m^3)	
	Poisson ratio		0.2 υ		
3^{50} . 4	12 13 10 -11 9 14	18 19 16 15 -17	20 -21 22 -24 25 26 23	-27 -30 28 29	

Table 1: Dimensional and material properties of the model.

Figure 5. FE model of the bridge.

To test the damage detection capability of the AHA algorithm, two simulated damage cases are created by reducing the cross-section of the damaged elements. In the first scenario, the cross-section area of the first element is reduced by 10%. In the second scenario, the crosssection area of the first element, the fourth element, and the seventh element are reduced by 10%, 12%, and 8%, accordingly. The results of the first eight natural frequencies are shown in [Table](#page-6-2) *2* below:

Model		$f_1(Hz)$ $f_2(Hz)$ $f_3(Hz)$ $f_4(Hz)$ $f_5(Hz)$ $f_6(Hz)$ $f_7(Hz)$ $f_8(Hz)$ $f_9(Hz)$				
Undamaged	1.418	5.661 12.692 18.639 22.452 34.861 49.821 55.971 67.220				
Scenario 1	1419	5.667	12.715 18.570 22.511 34.971 49.982 55.822 67.412			
Scenario 2	1.422	5.695	12.769 18.454 22.562 35.042 50.074 55.866 67.482			

Table 2: Natural frequencies of the three cases.

The objective function used in the problem has the following form:

$$
OF = \sum_{i=1}^{n} \frac{f_i^2}{\tilde{f}_i^2} = \sum_{i=1}^{n} (f_i - \tilde{f}_i)^2 / \tilde{f}_i^2
$$
 (6)

Where: n is the corresponding number of natural frequencies, f_i is the original-model frequency and \tilde{f}_i is the damaged-model frequency. The frequency is used as the parameter because it is sensitive to the changes in the structure behavior, which is usually a result of structural damage. To clarify the superior efficiency of the AHA, the authors compared it with the following optimization algorithms: CS, TLBO, PSO, and GA. The same input parameters are applied for all algorithms with population size $n_p = 200$ and the maximum number of iterations $n_{it} = 200$. The values of the input parameters are chosen to ensure all the proposed algorithms could have an unbiased result in solving the optimization problems, as the increase in these parameters would not affect the final optimized results. The algorithms are run on a computer with the following processor configuration: 12^{th} Intel® CoreTM i7-12700F 32GB RAM, NVIDIA GeForce 3060 RTX.

The results after running the algorithm for each hypothetical case are shown below.

Figure 6. Damage detection results in terms of location and severity of damage.

Figure 7. Best fitness obtained.

From the obtained results, it can be seen that for both the single and multiple damages cases, all the considered algorithms are able to locate and quantify structural damage correctly, with slight errors in the damage location for GA, PSO, and CS in the multiple damage scenario, while in terms of fitness convergence, TLBO, GA, PSO, AHA, and CS has the convergent speed in the decreasing order accordingly. In terms of computational time, AHA performs the fastest, followed by PSO, GA, CS, and TLBO, as shown in [Figure](#page-8-0) 8.

3.2.Chuong Duong truss span

In the second case study, a more complex bridge structure is considered. Vietnam's Chuong Duong Bridge (Figure 9) spans the Red River with a four-lane traffic road (No. 1A National Road) that has two lanes in the center for vehicles and buses and two more on either side for motorcyclists. The main structural system is a truss bridge consisting of 21 spans. Vietnamese engineers designed and constructed the main truss bridge from 10/10/1983 to 30/6/1985. The bridge is 1230 meters long and 19 meters wide overall. The top plated bar above and below is 600 mm wide, the top oblique bar is 640 mm, and the widths of the other diagonal bars and the vertical bar are 600, 460, 420, and 260 mm, respectively.

Experiencing a long operation time with overload, the traffic density on the bridge is ten times higher than the initial design. The steel truss bridge has contained different damages, such as warping of truss bars, corrosion of the main truss chord, upper chord, lower chord, and truss buttons.

Figure 9. General view of Chuong Duong bridge.

Finite element model

detailed in Figure 11. with the field vibration measurements' results using different accelerometers which is span is created (Figure 10) using MATLAB. The FE has been updated and correlated To assess the efficiency of the AHA algorithm, the Finite Element (FE) model of the 10th

Figure 10. Finite element model.

Figure 11. Accelerometers.

Some model parameters are as follows:

- The number of nodes: 45 represents truss nodes.
- as upper chord, lower chord, floor beam, diagonal, vertical, bracing, etc.; The number of elements: 146 – represents the elements of the structure such
- Boundary conditions: arrangement of bridge bearings according to the actual

structure.

- Materials: use steel material with elastic modulus $E = 2.10^{11}$ MPa, Poisson coefficient $v = 0.3$; density $DEN = 7850 \text{ kg/m}^3$
- Cross-section: including I, H, and box sections. The geometrical features of the section types are calculated and included in the model.

Damage detection validation

The parameter used as input to determine the algorithm's efficiency is the natural frequency because it is very sensitive to failures occurring on the bridge. To simulate damages due to the corrosion of the truss bars, cross-section deterioration is introduced. Two scenarios of damages are generated [\(Figure 12\)](#page-10-0) as shown below:

- Scenario 1 Single damage: Element no.1 reduced cross-section by 70%.
- Scenario 2 Multiple damages: Element no.1,4 reduced cross-section by 70% and 10%, respectively.

Figure 12. Damaged element's location.

The results of the first six natural frequencies are summarized in [Table](#page-10-1) 3:

The same objective function (6) is used for the damage detection of the bridge according to the first six natural frequencies. The same input parameters are applied for all algorithms with population size $n_p = 200$ and the maximum number of iterations $n_{it} = 200$. The

algorithms are run on a computer with the following processor configuration: $12th$ Intel® Core™ i7-12700F 32GB RAM, NVIDIA GeForce 3060 RTX.

The results after running the algorithm for each hypothetical case are shown below.

The results of the five algorithms in [Figure](#page-11-0) 13 show the single-damage problem is relatively accurate in both location and quantification of damage. For the multi-damage problem, only AHA gives correct damage location and level while CS, PSO, TLBO and GA gives additional minor false damages location and results.

Figure 14. Best fitness obtained.

[Figure 14](#page-11-1) shows that the convergence speed of CS in both cases is the slowest, and TLBO is the fastest. Convergence of AHA in single damage case starting from $70th$ iteration and $170th$ for the multiple damage case. The TLBO, PSO, and GA algorithms all converge quite quickly with large convergence slopes, but to achieve accuracy, these algorithms need to compute in subsequent iterations with more volume.

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It can be easily seen that the calculation time of both cases in [Figure 15](#page-12-0) of AHA is the fastest of the five algorithms and is similar to each other. PSO and GA have fast convergence, but it takes 60% more time than AHA to find the final result. Meanwhile, CS and TLBO take twice as much computation time as AHA to detect the location and extent of the damage.

4. CONCLUSION

In this study, the performance evaluation of Artificial Hummingbird Algorithm is assessed to solve the structural damage identification problems for two case studies of bridge structures in Vietnam. The algorithm is compared with four others widely used algorithms-CS (Biobased), TLBO (Human-based), PSO (Swarm-based), and GA (Evolutionary-based) for effectiveness. The result indicates that, in the problem of generated single and multiple damage identification, AHA is able to provide accurate results in both finding the location and level of damage. Moreover, the computational cost of AHA is much superior to the other algorithms while maintaining a competitive convergence speed. It shows that AHA works effectively in solving optimization problems for damage identification of structures and can be used to enhance the effectiveness of the current SHM system.

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