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# THEORETICAL CALCULATION OF BENDING CAPACITY OF A STEEL BEAM - ULTRA HIGH PERFORMANCE CONCRETE SLAB COMPOSITE GIRDER

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**Abstract.** The composite girder, which combines a steel beam with an ultra-high-performance concrete (UHPC) slab, has gained significant attention in recent years as a new type of bridge structure. However, accurately estimating the bending capacity of such girders remains a challenge, as practical methods are limited. In this article, the authors propose a theoretical approach based on the Euler-Bernoulli beam theory to determine the bending capacity of composite girders. This approach considers the assumptions of plane sections remaining plane and infinitesimal strains during bending. By applying this theoretical approach, the authors derive a formula that allows engineers to calculate the bending capacity of the composite girder. The formula takes into account the dimensions and properties of both the steel beam and the UHPC slab. The derived formula serves as a valuable tool for evaluating the structural behavior and performance of composite girders. To validate its accuracy, the authors compare the results obtained from their calculations with numerical simulations of composite girder failures caused by bending. The close agreement between the theoretical calculations and the numerical simulation results confirms the reliability and applicability of the proposed formula. This research significantly contributes to the field of composite girder design by providing a practical and reliable method for estimating the bending capacity of steel beam-UHPC slab composite girders. The proposed theoretical approach, validated through numerical simulations, offers valuable insights for the design and optimization of these composite girders in various engineering applications.
**Keywords:** composite girder, bending capacity, ultra high-performance concrete

## 1. INTRODUCTION

Studies on the application of UHPC in bridge structures, both in design and repair aspects, have been widely deployed in recent times (see [1-3]). In steel girder structures, UHPC is being used as deck slabs in composite steel beams to replace conventional steel girder structures. Figure 1 to 4 show examples of steel - UHPC composite girder being used to replace conventional composite beams (see [4-6]).

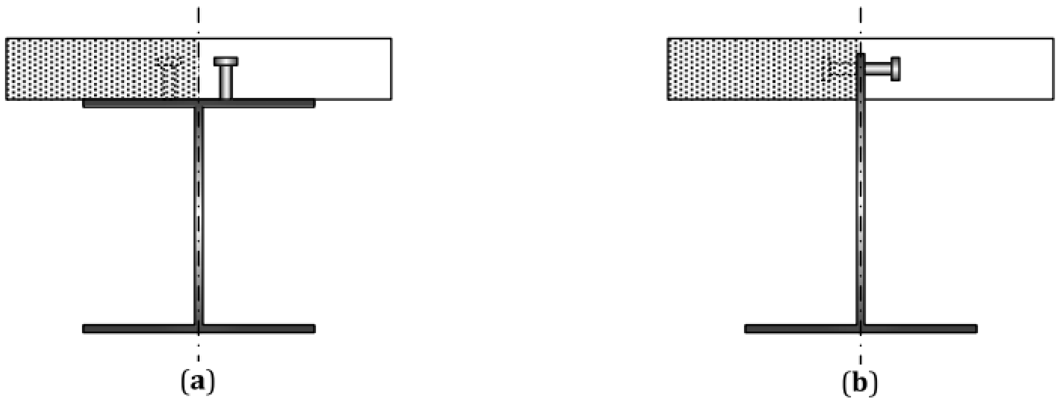



Figure 1. (a) conventional composite girder; (b) composite beam with Inverted-T steel girder and UHPC slab [4].

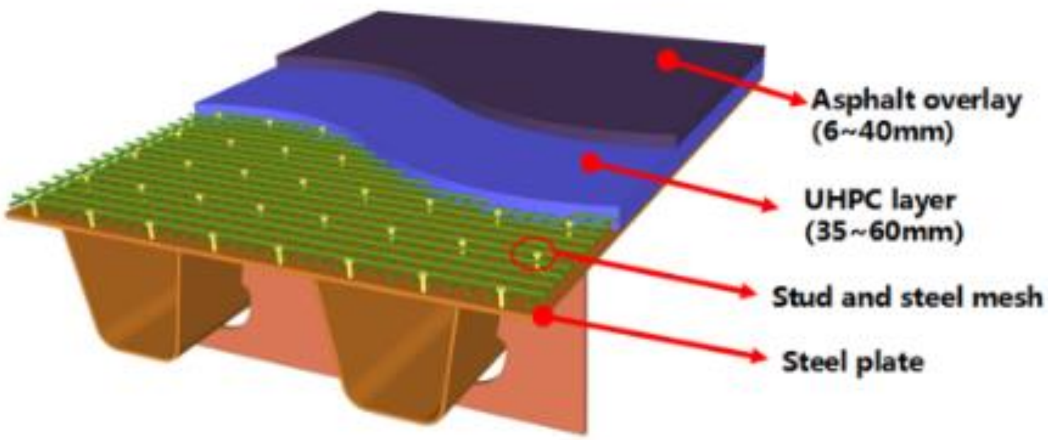



Figure 2. composite (steel+ UHPC) lightweight deck system [5].

It can be argued that there are several types of steel beam-UHPC slab composite girders that offer significant advantages in terms of working efficiency. However, there is currently no comprehensive theoretical approach available to determine the flexural resistance of these girders. Therefore, it is necessary to develop a theoretical calculation procedure based on beam theory to determine their bending capacity, which would be useful for engineers. The main focus of this paper is to propose a theoretical equation for the ultimate bending capacity of steel beam-UHPC slab composite girders. The theoretical results will be compared with the analysis results obtained through non-linear finite element analysis to assess the practical usefulness of this theoretical equation.

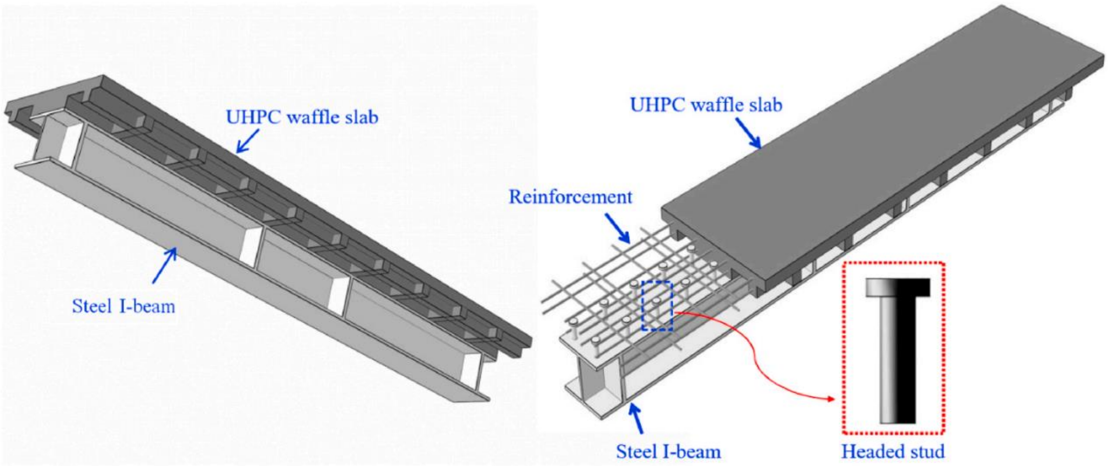



Figure 3. steel-UHPC composite beams with waffle slab [6].

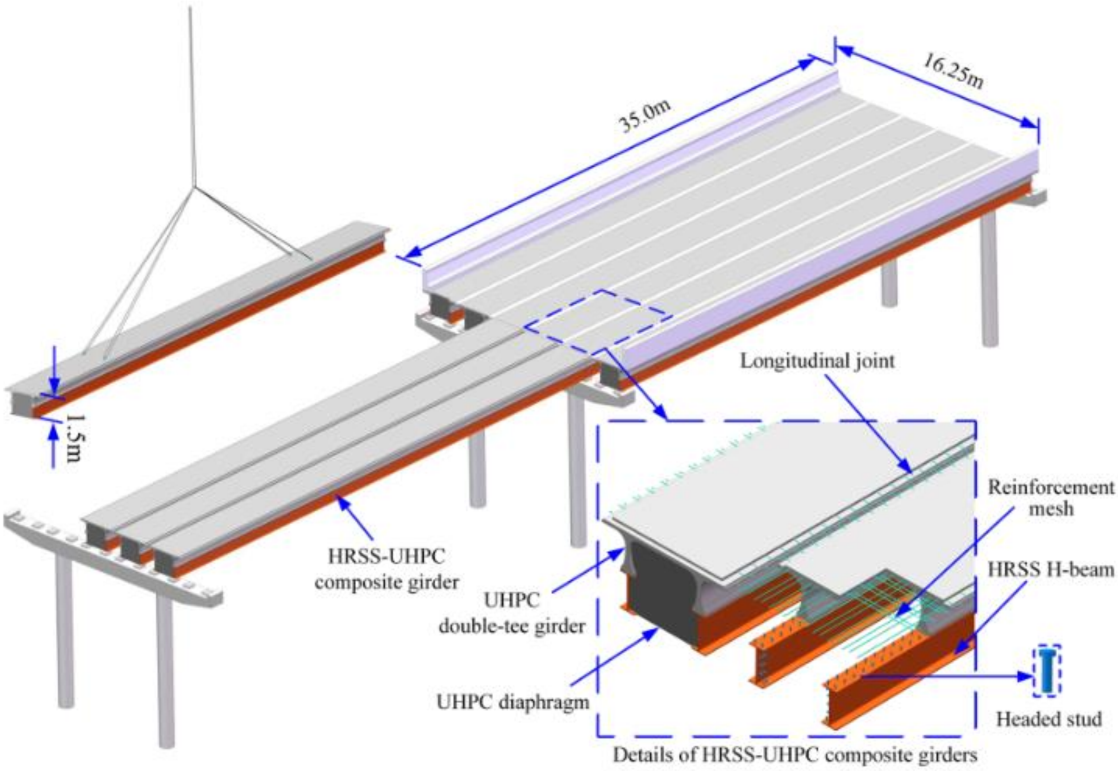



Figure 4. Hot rolled shape steel-ultrahigh performance concrete composite beam [7].

## 2. THEORETICAL CALCULATION OF BENDING CAPACITY OF STEEL BEAM-UHPC SLAB COMPOSITE GIRDERS

Ultra-high-performance concrete (UHPC) is a type of concrete that possesses exceptionally high compressive and tensile strength compared to conventional concrete. Numerous authors have extensively studied the behavior of UHPC under compression and tension, both experimentally and theoretically (refer to [8-9] for further details). Design standards for structures in various countries, including France, Austria, and Korea ([10]), have incorporated a standard model for UHPC to facilitate design calculations. Figure 5 illustrates the widely accepted material model of UHPC, which is commonly utilized in calculations (see [11]).

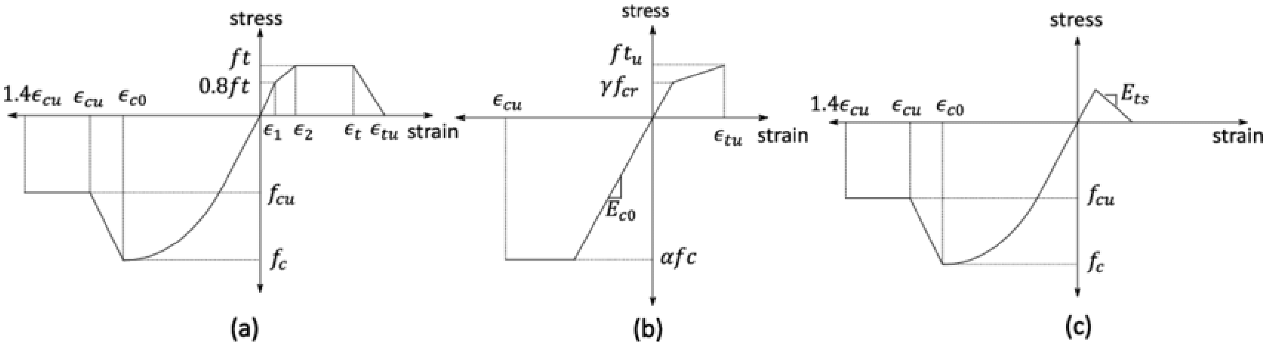



Figure 5. Constitutive model for (a) UHPC; (b) Idealized UHPC; and (c) Conventional concrete [11].

For the steel beam and UHPC slab composite girder, we propose using an "idealized" model for UHPC (Figure 5b) and an elastoplastic model for steel (Figure 6) to develop the calculation equations.

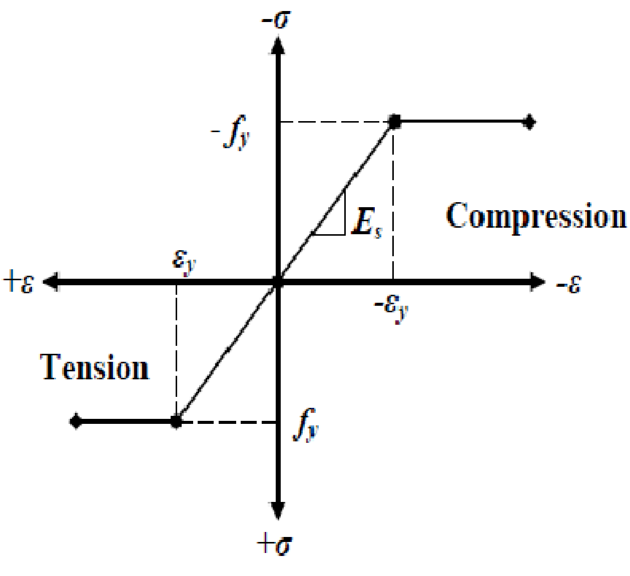
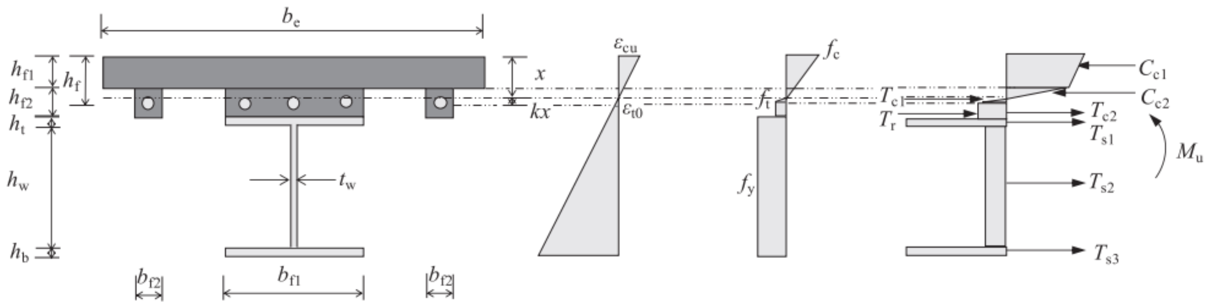


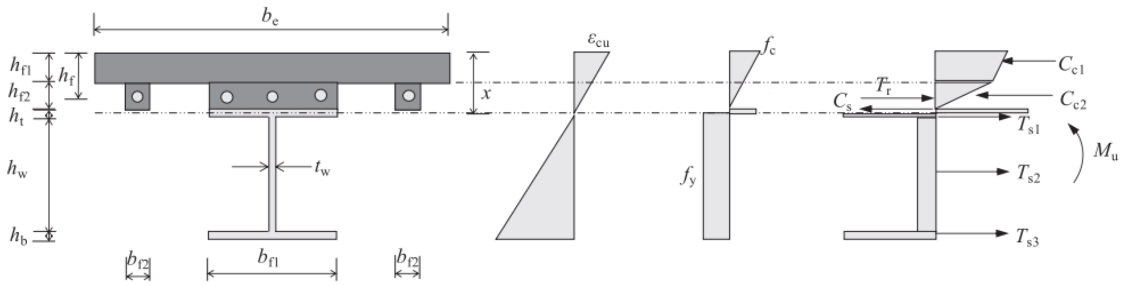

Figure 6. Elastoplastic model for steel [11].

In beam theory, assuming the infinitesimal strain theory and the Euler-Bernoulli hypothesis, which states that plane sections before bending remain plane after bending, the relation between normal stress and normal strain in the cross-section can be calculated. Figure 7 shows the general case of this relation.

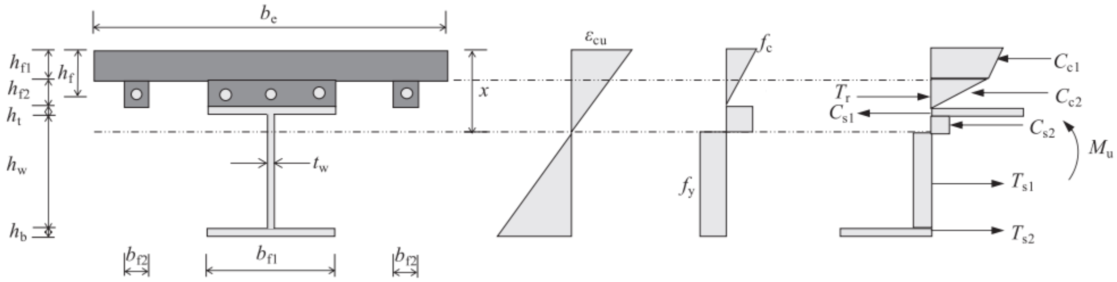
When the steel section is transformed into a standard I-shape, the ultimate bending moment of the section is defined as the moment that causes the entire tensile steel beam to yield (normal stress reaching the yield strength $F_y$), and the top of the slab to compress up to the compressive strain limit ($\varepsilon_u$). This state is achieved based on one of the following cases: (1) the plastic neutral axis passes through the UHPC slab, (2) the plastic neutral axis passes through the top flange of the steel beam, or (3) the plastic neutral axis passes through the web of the steel beam. In each case, the position of the plastic neutral axis is determined by the condition that the absolute value of the plastic tensile force is equal to the destructive compressive force (as there is no axial force in the beam cross-section) (refer to Figure 8). The nominal value of the "ultimate moment" for the beam is determined using a formula commonly employed for composite beams, as depicted in Figure 9.




(a) Case 1. Plastic neutral axis passes through the UHPC slab.




(b) Case 2. Plastic neutral axis passes through top flange.




(c) Case 3. Plastic neutral axis passes through the web

Figure 7. Normal stress – strain relation in a steel beam – UHPC slab composite girder for different position of plastic neutral axis [11].

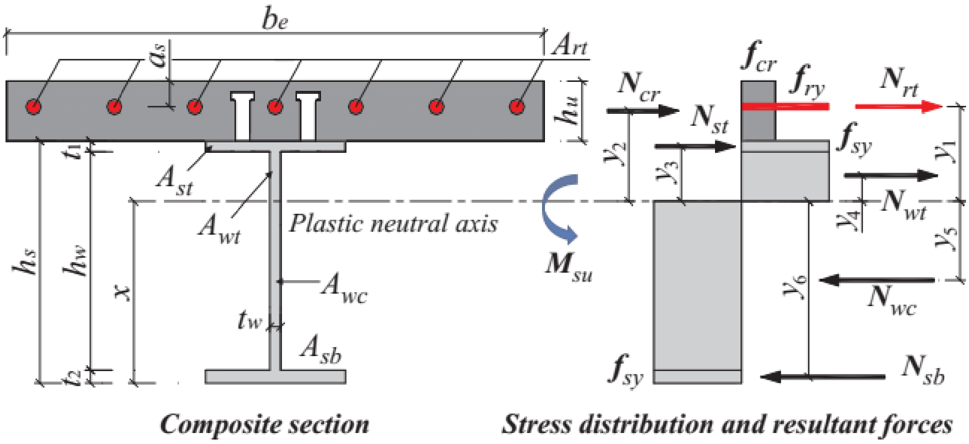



Figure 8. Yield force in different parts of the steel beam – UHPC slab composite girder for the case the plastic neutral axis in web.

In each case, the bending capacity can be calculated by the equation

$$M_n = \sum (N_i y_i) \quad (1)$$

In which: $N_i$ is the ultimate normal force for a particular part, $y_i$ is the distance from the consider part to the plastic neutral axis.

## 3. COMPARISON OF THEORETICAL CALCULATION WITH NON-LINEAR NUMERICAL ANALYSIS

To evaluate the accuracy of the theoretical formula, the bending capacity calculated by the theoretical formula will be compared to the results obtained from numerical simulation for an example of a steel beam - UHPC slab composite girder, as shown in Figure 10.

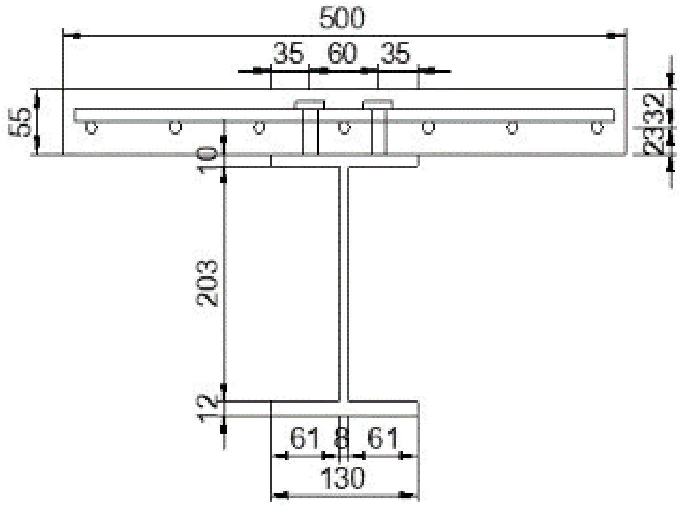



Figure 9. Example of steel beam – UHPC slab composite girder.

In this example, the mechanical properties of high performance steel is employed, in which the yield stress of steel is $F_y = 418MPa$, ultimate strength $F_u = 540MPa$, ultimate strain

$\varepsilon_u = 0.2$, yield strain $\varepsilon_y = 0.02$. For UHPC, the compressive strength is $f_c$ =120 MPa, ultimate compressive strain of UHPC is $\varepsilon_{cu} = 0.02$, tensile strength $f_{ku}$ = 7MPa. When ignoring the contribution of reinforcement bars in the slab, the nominal value of bending capacity for the example cross-section can be estimated as described in the figure 11. In which, the plastic neutral axis passes through the UHPC slab, with a distance of 32.5mm to the top of the slab. In the analysis of the composite girder, the ultimate moment of the cross section is determined by the failure modes of its components. Specifically, the ultimate moment is reached when the compressive part of the UHPC slab experiences compressive failure, and the tensile part of both the UHPC slab and the steel beam reaches their ultimate tensile forces. The UHPC slab, known for its exceptional compressive strength, undergoes compressive failure when it reaches its maximum compressive stress capacity. This failure occurs at the top of the slab, where it is subjected to the highest compressive forces. On the other hand, the tensile part of the UHPC slab and the steel beam experience ultimate tensile forces, leading to their failure. The UHPC slab is capable of withstanding significant tensile forces before reaching its ultimate capacity, while the steel beam, known for its high tensile strength, also exhibits a corresponding ultimate tensile force. By considering the failure modes of these components, the ultimate moment of the composite girder can be accurately determined. This moment represents the maximum load that the girder can sustain without experiencing structural failure. Understanding the behavior and limits of the composite girder is crucial for its design and ensuring its structural integrity in practical applications. In figure 11, $F_{c1}$ denotes the ultimate compression on UHPC in ultimate state of the cross section while $F_{t1}$ is the ultimate tensile force of UHPC part which subjected to tension and $F_{t2}$ is the ultimate tensile force of the steel part.

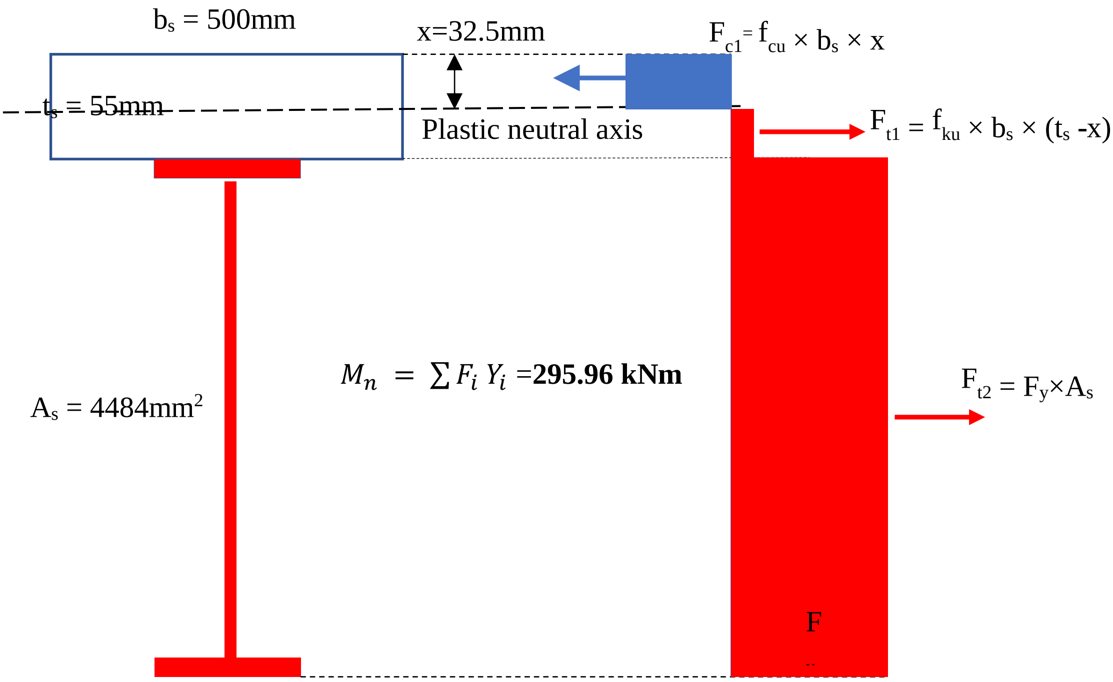



Figure 10. Theoretical calculation of bending capacity of the steel-UHPC composite cross section.

It is important to note that non-linear finite element analysis can provide a more accurate prediction of the flexural resistance of the composite girder compared to the mentioned theoretical formula since the assumptions such as the plane strain hypothesis is not necessary to be taken into account. Therefore, in this paper, the described composite girder is modeled by a FEA program Abaqus for a four-point bending test with force points located 0.65m from the beam end as shown in figure 12.

Based on the numerical analysis, it can be inferred that the composite girder fails when the deflection reaches 7.8mm. At this point, the bottom fiber of the steel girder yields and the top UHPC reaches compressive failure (as shown in Figure 13). The ultimate compression at the four-point bending test is 510kN (as shown in Figure 14), which is equivalent to the ultimate bending capacity of Mn = 510kN × 0.65m = 312kNm.

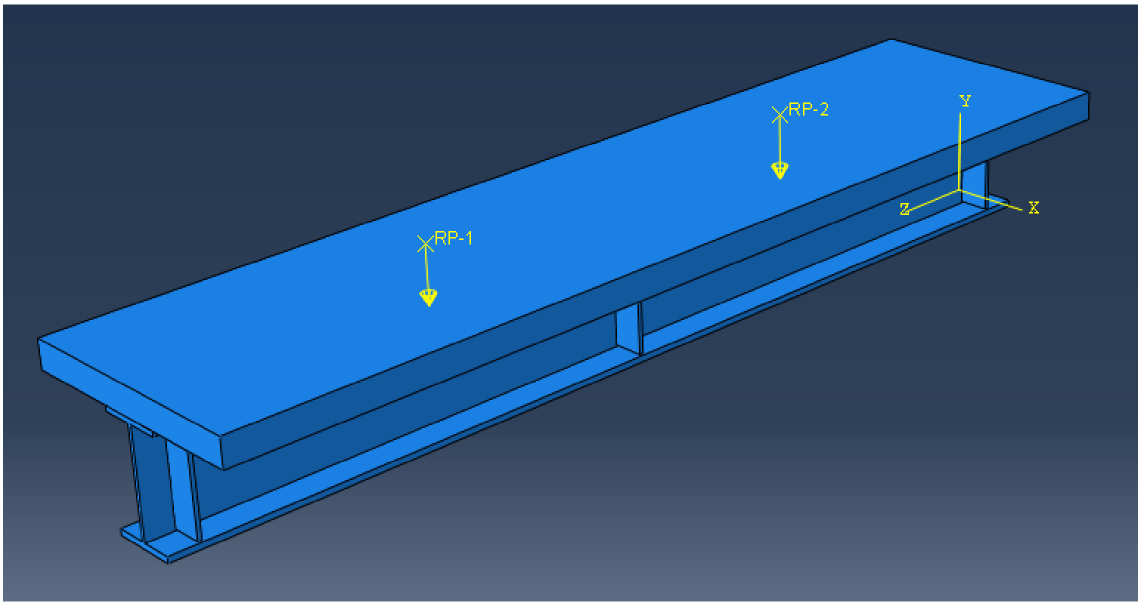



Figure 11. Numerical simulation for four-point bending test of the composite girder.

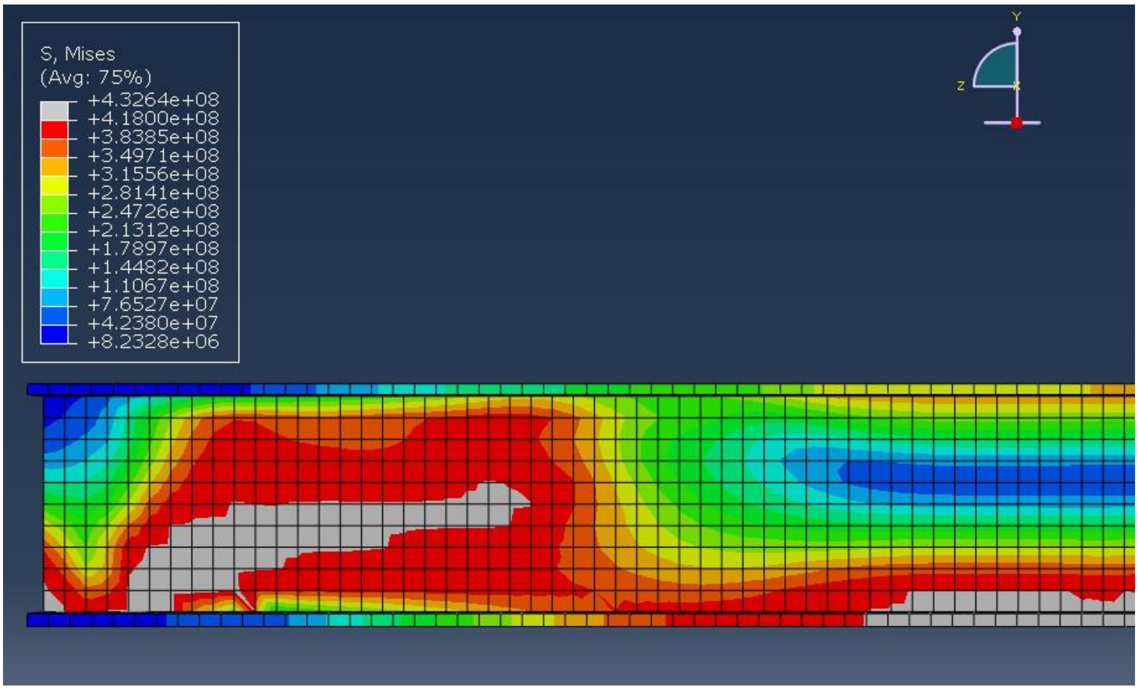



Figure 12. Stress- strain condition in a half of girder at ultimate failure state.

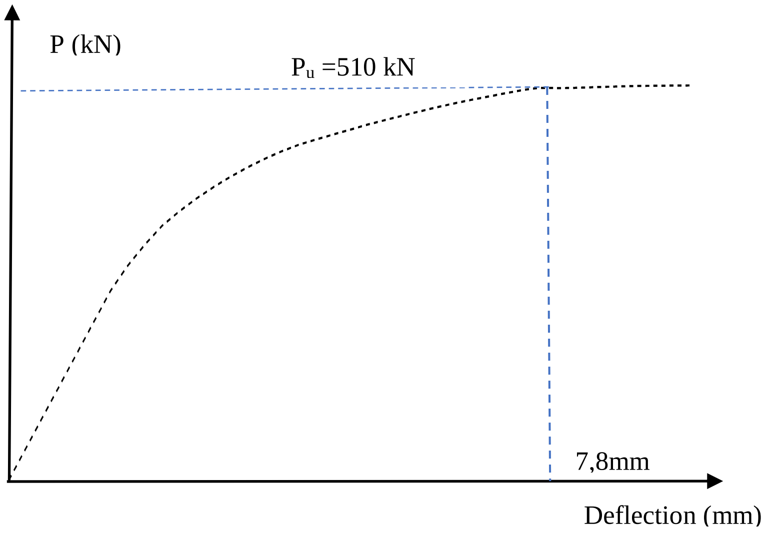



Figure 13. Relation between the applied force and the deflection of the composite girder.

By comparing the theoretical calculations based on beam theory and the results obtained from 3D solid element modeling using the nonlinear theory of materials, it was observed that the bending capacity calculated by the theoretical formula is 94.86% of the modeled value. This suggests that the theoretical formula is a safe and acceptable method for design purposes.

## 4. CONCLUSION AND DISCUSSION

In conclusion, the research study highlights the applicability of the theoretical formula based on beam theory for evaluating the flexural strength of a steel beam - UHPC slab composite girder. The formula shares similarities with the conventional steel beam - concrete slab composite girder, making it a practical tool for engineers and designers.

To validate the accuracy of the theoretical formula, a comparison was made between the calculated bending resistance using the formula and the limiting moment obtained from numerical simulations that consider the non-linear behavior of materials. The findings reveal that the theoretical formula provides results with acceptable accuracy.

The implications of these results are significant, as they provide reassurance regarding the use of the theoretical formula in design processes. Engineers can rely on the formula to estimate the flexural strength of steel beam - UHPC slab composite girders, allowing for more efficient and reliable structural designs.

By employing the theoretical formula, designers can streamline their workflow and reduce reliance on extensive numerical simulations, saving time and resources. However, it is important to note that the theoretical formula should be used in conjunction with appropriate safety factors and considerations to ensure the overall structural integrity and safety of the composite girder.

Overall, the study contributes valuable insights into the behavior of steel beam - UHPC slab composite girders and provides a practical and reliable method for evaluating their flexural strength.

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