



## ANALYTICAL TRUSS MODEL FOR CONCRETE BEAMS REINFORCED WITH FRP BARS

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**Abstract.** A truss model supplementing the concrete contribution is introduced in this paper to predict the shear strength of concrete beams reinforced with various types of FRP bars. The contributions from truss and direct strut mechanisms are considered in the analytical model. The truss model which has struts at various angles considering concrete contribution is derived in this paper. The concept of equivalent transverse FRP reinforcement is applied to integrate the concrete contribution into the proposed truss model. The shear strengths of concrete and FRP transverse reinforcements in the proposed model are calculated based on El-sayed et al. and CSA S806, respectively. The validity and applicability of the proposed model are evaluated by comparison with available experimental database, which consists of concrete beams reinforced with glass, carbon, aramid and basalt FRP bars. The comparison has shown a good correlation between the experimental data and analytical results. The proposed model is also compared with ACI 440.1R and CSA S806's shear strength models and the better correlation is found. The results from this research shows that the properly-treated truss analogy can be used to assess the shear strength of concrete beams reinforced with various types of FRP bars.

**Keywords:** FRP bars; concrete beams; strut-and-tie; beam actions; arch actions; shear strength.

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### 1. INTRODUCTION

The role of fiber-reinforced polymer (FRP) has long been recognized by the researchers' community as an alternative material for conventional steel reinforcements in concrete

structures. The superior durability of FRP makes them attractive for concrete structures exposed to harsh environments such as de-icing and marine ones. The further advantages of these special materials over conventional steel reinforcements could be named: high strength, high strength-to-weight ratio, resistance to chemical attacks and controllable thermal expansion. However, the stress-strain behaviors of FRP bars are very much different from the ones of conventional steel bars, especially the modulus of elasticity. These differences could result in the different shear strength of concrete members, which is one of the most important design properties for structural concrete members. The failure of concrete members in shear could lead to catastrophic consequences. Therefore, it is obvious that a reliable analytical model to estimate the shear strength of concrete members reinforced with FRP bars is essential.

Different approaches have been tried by several researchers (such as El-Sayed *et al.*[1]) to predict the shear strength of concrete structures reinforced with FRP bars during the last decades. This had resulted in several design guidelines for concrete structures reinforced with FRP bars such as the CSA S806 [2], ACI 440.1R [3], ISIS [4] and JSCE [5] design codes. The shear strength provisions in these guidelines for concrete structures reinforced with FRP bars were developed based on the commonly-used formula ( $V_s+V_c$ ) for conventional steel reinforced concrete structures. Several modification factors accounting for the significant difference in the modulus of elasticity between FRP and conventional steel reinforcements had been proposed by these codes. Most recently El-Sayed *et al.*[1] had indicated that the ACI 440.1R [3] shear strength equation significantly underestimates the concrete shear strength of concrete beams reinforced with FRP bars. El-Sayed *et al.*[1] had proposed some modifications to the ACI 440.1R [3]'s shear strength equation to overcome this problem. Hoult *et al.*[6] had also tried to predict the shear strength of concrete members reinforced with FRP bars. The shear strength equation in Hoult *et al.*[6]'s research was derived based on the modified compression field theory (MCFT). Several researches had been done to predict the shear strength of concrete members reinforced with FRP bars as mentioned previously, however there are no strut-and-tie models be applied to study the shear behavior of these structures. The strut-and-tie models have proved its accuracy and superiority in modeling the concrete structures failing in shear manner. Therefore, further researches should be focused on this area.

Many researchers (such as Bresler and MacGregor [7], Li and Tran [8], Tran [9]) had made significant contributions to the development of strut-and-tie models of reinforced concrete (RC) beams subjected to shear and flexure. One of the earliest and most important contributions came from Bresler and MacGregor's study [7]. Based on a classical truss model, Bresler and MacGregor [7] explained the failure mechanisms and analyzed the shear behavior of RC beams failed in shear. This study provides an initial concept of tension and compression chords, compression strut and tension tie members. The complex stress fields inside concrete members failed in shear could be simplified by discrete members; namely tension and compression chords, strut and tie members. The tensile longitudinal reinforcements are considered as tension chords, whereas the compression chords are contributed by the compression zone and compression longitudinal reinforcements. The longitudinal compression and tension chords are joined together by tension ties and compressive struts. The tension ties and compressive struts come from transverse reinforcements and concrete compressive stress field, respectively.

Based on the previous conceptual model developed by Bresler and MacGregor [7], Li and Tran [8] had proposed an analytical truss model for RC beams failed in shear. A truss model with struts at various angles had been developed in their study (2008). The concept of equivalent transverse reinforcement had been applied in Li and Tran's study [8] to implement the concrete

contribution in shear. Both beam and arch actions had been modeled. The developed model was then verified against the RC beams failed in shear with an aspect ratio from 2 to 3, where both beam and arch actions are prevailed. The truss model obtained from their study had given satisfactory analytical results as comparing to the experimental evidences.

Most recently, Tran [9] had further developed Li and Tran's model [8] for RC columns with non-seismic details subjected to lateral loadings. The concept of equivalent transverse reinforcement was also applied in this study. This had resulted in a very good estimation of shear strengths of RC columns with non-seismic details subjected to lateral loadings.

The traditional truss model developed by Bresler and MacGregor [7] had proved its capability in not only conceptually visualizing the stress field in concrete members reinforced with traditional steel bars failed in shear but also accurately analyzing the shear strength of these members. Hence, additional researches should be concentrated on this area to further develop this method for other types of concrete members failed in shear such as concrete structures reinforced with FRP bars. This is the main focus of this paper. The successful strut-and-tie model previously proposed by Li and Tran [8] for conventional RC beams failed in shear will be applied and modified in this paper to predict the shear strength of concrete beams reinforced with FRP bars. The concrete contribution in shear will be accounted for based on the concept of equivalent transverse FRP reinforcements. The significant difference in the modulus of elasticity between FRP and conventional steel reinforcements is also considered in the proposed model.

There are two parts in this paper. The shear strength models following the commonly-used and well-known design codes are reviewed and the shortcomings of these models are discussed in the first part. The derivation of the new strut-and-tie model for concrete beams reinforced with FRP bars is reported in the second part. In this part, the improved performance of the proposed model is identified by comparison with the ACI 440.1R [3] and CSA S806 [2]'s shear strength models. Further parametric studies based on the proposed model for concrete beams reinforced with FRP bars are also presented in this part of the paper.

## **2. PREVIOUS DESIGN EQUATIONS FOR SHEAR STRENGTH OF CONCRETE BEAMS REINFORCED WITH FRP BARS**

The shear strength equations for concrete beams reinforced with FRP bars in the commonly-used and well-known design codes, specifically the ACI 440.1R [3] and CSA S806 [2] codes are reviewed in this part of the paper. The shortcomings and improvements in these design codes are discussed in details. Hence, the needs for a realistic modeling method for concrete beams reinforced with FRP bars with an aspect ratio of from 2 to 3 are highlighted at the end of this part of the paper.

### **2.1. ACI 440.1R [3]**

The ACI 440.1R [3] on the nominal shear strength of concrete beams reinforced with FRP bars suggests the following additive equation:

$$V_n = V_c + V_f \quad (1)$$

Where the concrete shear capacity of members using FRP as main reinforcement,  $V_c$  is calculated as follows:

$$V_c = \frac{2}{5} k \sqrt{f'_c} b_w d \quad (2)$$

where  $f'_c$ ,  $b_w$ ,  $d$  are the compressive strength of concrete, width of beams and distance from extreme compression fiber to centroid of tension reinforcement, respectively.

The difference between traditional steel and FRP longitudinal reinforcement in the shear strength of concrete is reflected through the factor  $k$ . Due to a lower axial stiffness of FRP reinforcement as compared with steel one, the concrete beams reinforced with FRP longitudinal reinforcements will have wider crack widths leading to a smaller compression zone as comparing with the ones with steel reinforcements. Therefore, a lower shear strength contributed by the concrete is expected in the concrete beams reinforced with longitudinal FRP bars. The factor  $k$  is calculated as:

$$k = \sqrt{2\rho_F n_f + (\rho_F n_f)^2} - \rho_F n_f \quad (3)$$

where  $\rho_F, n_f$  are the fiber-reinforced polymer reinforcement ratio and ratio of modulus of elasticity of FRP bars to modulus of elasticity of concrete, respectively.

The shear contribution of FRP stirrups through truss mechanism is:

$$V_f = \frac{A_{fv} f_{fv} d}{s} \quad (4)$$

where  $A_{fv}$  is the amount of FRP shear reinforcement within spacing  $s$  and  $s$  is stirrup spacing.

To control the widths of shear cracks and avoid failure at the bent portion of the FRP stirrup, the design stress level in the FRP shear reinforcement is limited as:

$$f_{fv} = 0.004E_f \leq f_{fb} \quad (5)$$

where  $E_f, f_{fb}$  are the modulus of elasticity of FRP bars and strength of bent portion of FRP bars, respectively.

## 2.2. CSA S806 [2]

Similar to the ACI 440.1R design code [3], an additive equation is used to calculate the nominal shear strength following the CSA S806 design code [2]. An upper limit of  $0.22f'_c b_w d_v$  is specified for the nominal shear strength according to the CSA S806 design guideline [2].

The design shear strength following the CSA S806 code [2] considers not only the effects of longitudinal reinforcement rigidity as similar to the ACI 440.1R code [3] but also the effects of moment to shear ratio, arch action and size effect. This shows the improvements of the CSA S806 code [2] as comparing with the ACI 440.1R code [3]. These improvements had resulted in more accurate predictions of the shear strengths of concrete beams reinforced with FRP bars as mentioned in the previous research (Razaqpur and Spadea [10]). The CSA S806 design code [2] specifies the concrete contribution of FRP-reinforced concrete members as follows:

$$V_c = 0.05\lambda\phi_c k_m k_r k_a k_s (f'_c)^{1/3} b_w d_v \quad (6)$$

Where  $\lambda$  is the concrete density factor,  $\phi_c$  is material resistance factor and  $d_v$  is the effective depth of beams.

Where  $k_m, k_r, k_a, k_s$  are the factors accounting for the effects of moment to shear ratio, longitudinal reinforcement rigidity, arch action and size effect, respectively. These factors are calculated as follows:

$$k_m = (V_f d / M_f)^{1/2} \quad (7)$$

$$k_r = 1 + (E_f \rho_f)^{1/3} \quad (8)$$

$$k_a = (2.5 V_f d) / M_f \text{ for } a/d < 2.5 \quad (9)$$

$$k_s = \left( \frac{750}{450 + d} \right) \quad (10)$$

The size effect factor,  $k_s$  is only applied for members with an effective depth greater than 300 mm and with less transverse reinforcement than the minimum requirement as specified in the design code [2].

In the CSA S806 code [2], a further improvement is implemented in this code as comparing to the ACI 440.1R code [3] in calculating the shear strength of FRP stirrups. The variable truss angle approach is applied in the CSA S806 code [2] instead of the constant angle of 45° degree one specified in the ACI 440.1R code [3]. It is the true behavior of concrete structures with variable inclined crack angles. The inclined crack angles changes depending on the loading conditions and the properties of concrete members. This improvement could be one of the factors contributing to better predictions of shear strengths of concrete beams reinforced with FRP bars produced by the CSA S806 code [2] as comparing with the ACI 440.1R code [3] as mentioned in the previous research (Razaqpur and Spadea [10]).

The shear contribution of FRP stirrups through the truss mechanism as defined in the CSA S806 code [2] is:

$$V_f = \frac{A_{fv} f_{fv} d_v}{s} \cot \theta \quad (11)$$

In which the truss angle  $\theta$  is given as:  $\theta = 30 + 7000 \varepsilon_1$ . The average longitudinal strain at mid-height of the considering section is:

$$\varepsilon_1 = \frac{\frac{M_f}{d_v} + V_f + 0.5 N_f}{2 E_f A_f} \quad (12)$$

where  $N_f$ ,  $M_f$ ,  $V_f$  are the factored axial force, moment and shear force acting on the considering section.

The design stress level in the FRP shear reinforcement following CSA S806 code [2] is limited as:

$$f_{fv} = \min\{0.005 E_f, 0.4 f_{Fu}, 1200 \text{ MPa}\} \quad (13)$$

where  $f_{Fu}$  is design tensile strength of FRP bars.

### 2.3. Shortcomings of design equations for shear strength of FRP reinforced concrete beams following the existing design codes

As reviewed in the previous part, the design equations for shear strength in the major guidelines for concrete structures reinforced with FRP bars were developed based on the well-known additive formula ( $V_s + V_c$ ), which is the truss-like analogy. These methods are easy to use. However only beam actions are considered in the design equation of the ACI 440.1R code [3]. This may be the true behavior for concrete beams with an aspect ratio of higher than 3. For concrete beams with such a high aspect ratio, the beam action is predominant and there is no arch action forming between the point of the applied load and the support as shown in Fig. 1.

On the other hand, for concrete beams with an aspect ratio from 2 to 3, where both beam and arch actions are prevailed (Li and Tran [8]), the use of the equations based on only beam action could lead to inaccurate and conservative results. The CSA S806 code [2] had modified this shortcoming in the ACI 440.1R code [3] by implementing a factor ( $k_a$ ) to include the arch actions for concrete beams with an aspect ratio of less than 2.5. However, the modification of the traditional formula developed based on beam actions does not reflect the true behavior of concrete beams with an aspect ratio from 2 to 3, where both beam and arch actions are prevalent. In the next part of this paper, a strut-and-tie method, where both beam and arch actions are modelled, will be developed for concrete beams reinforced with FRP bars with an aspect ratio from 2 to 3 to predict the shear strength of such beams.

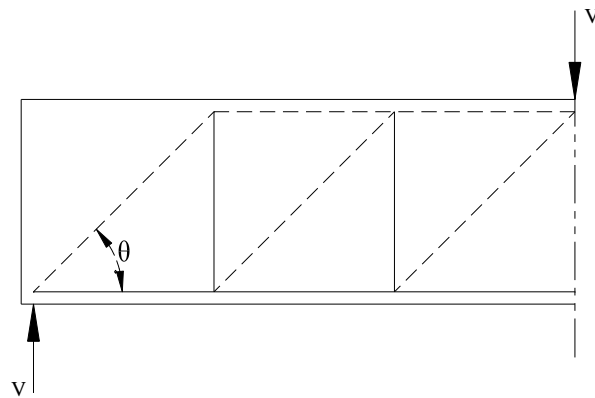


Fig. 1 Beam action in concrete beams reinforced with FRP bars.

### 3. PROPOSED SHEAR STRENGTH MODEL

#### 3.1. Proposed model

Fig. 2 showed a graphical presentation of the overall configuration of the proposed strut-and-tie model for concrete beams reinforced with FRP bars with an aspect ratio from 2 to 3. The compressive and tensile members are shown in dotted and solid lines, respectively. Similar to the Li and Tran's truss model [8] proposed for RC beams, the height of the proposed model is standardized and assumed to be constant along the shear span of concrete beams.

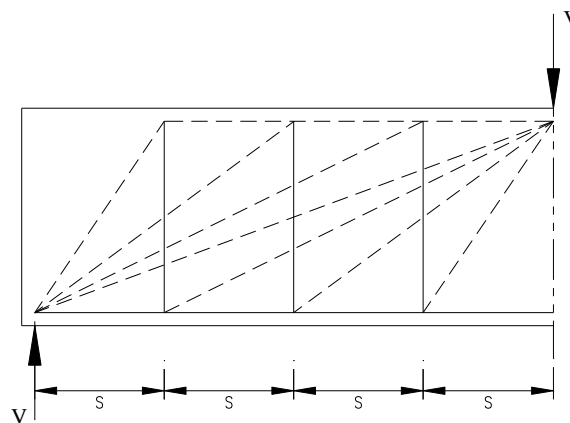


Fig. 2 Proposed strut-and-tie model for beams accounting for both beam and arch actions.

As discussed previously for beams with an aspect ratio from 2 to 3, both beam and arch actions contribute their parts in resisting the applied shear force. As shown in Fig. 2 the arch

action is modelled by a direct compressive member connected between the location of applied loads and the support. This member resists a part of the applied shear force through the compressive strength of concrete. The beam action transfers the applied shear force through the inclined concrete compressive struts with variable angles of inclination; FRP ties; compressive and tensile chords at top and bottom of the model, respectively. These members form a statically indeterminate strut-and-tie model with struts at various angles. In Li and Tran's study [8], the variable angle of inclination of concrete struts is achieved by dividing the shear span into four equal portions. This conceptual strut-and-tie model will be applied in this research for the concrete beams reinforced with FRP bars.

### 3.2. Properties of members in the proposed model

The overall configuration of the proposed truss model had been discussed in the previous part. There are four main members in the proposed model, namely tensile and compressive chords, inclined compressive struts and tensile ties. The properties and strengths of these members will be discussed in details in this part of the paper.

#### *Tensile and compressive chords*

The locations of tensile and compressive chords are assumed at the centers of FRP longitudinal reinforcements and equivalent concrete stress block, respectively when analyzing the section for the nominal flexural strength. The strength of the tensile chords is defined as  $A_{ffu}$ . The tensile capacity of concrete is ignored in calculating the strength of the tensile chord. Whereas the strength of compressive chords is given as  $0.85f'_c b\beta_1c$  following the ACI 440.1R code [3]. Because of a significant lower compressive strength than tensile strength in FRP reinforcements, the contribution of FRP bars in the compressive chord is ignored as recommended by the ACI 440.1R code [3]. The effective depth of the compressive chords is defined as equal to the depth of equivalent stress block,  $\beta_1c$ .

#### *Inclined compressive struts*

The actual stress field inside the FRP reinforced beams could be represented by the inclined compressive struts as shown in Fig. 2. At failure, there are significant cracks across the shear span. To cater for these cracks, a compressive strength reduction factor of 0.4 is applied in Li and Tran's proposed model [8] for these inclined compressive struts following Schlaich and Schafer's suggestions [11]. In this proposed model, the strength reduction factor in the latest concrete design code EC2 [12] is applied.

$$v_1 = 0.6(1 - f'_c/250) \quad (14)$$

In Li and Tran's proposed model [8], the effective depths of the inclined compressive struts are determined from their geometrical consideration at the mid-height of the strut. This method of determining the effective depth is not consistent with the use of strength reduction factor. Because severe cracks normally occur at the bottom half of the beams. Therefore, the effective depths of the inclined compressive struts are estimated as shown in Fig. 3.

#### *Tensile ties*

The locations of the tensile ties are determined similarly to Li and Tran's model [8] as defined previously. Both FRP bars and concrete contribute to the tensile capacity of the tie members. The tensile capacity of the FRP bar is defined as  $A_{fv}f_{fv}$ . Where  $f_{fv}$  is defined following the CSA S806 code [2].

El-Sayed *et al.* [1] indicated that the ACI 440.1R shear strength equation [3] significantly underestimates the concrete shear strength of concrete beams reinforced with FRP bars. Some modifications to ACI 440.1R's shear strength equation [3] had been proposed by El-Sayed *et al.* [1] to better predict the concrete shear strength of these beams. In this proposed strut-and-tie model, the simplified concrete contribution ( $V_c$ ) proposed by El-Sayed *et al.* [1] is used to determine the concrete contribution to tie members. This concrete contribution is converted to an equivalent FRP stirrups with a known tensile strength of FRP for shear design and then implemented to the strut-and-tie model. The concrete shear strength equation proposed by El-Sayed *et al.* [1] is as follows:

$$V_c = 0.037 \left( \frac{\rho_f E_f \sqrt{f'_c}}{\beta_1} \right)^{1/3} b_w d \leq \frac{\sqrt{f'_c}}{6} b_w d \quad (15)$$

Where  $\beta_1$  is a function of the concrete compressive strength:

$$0.85 \geq \beta_1 = 0.85 - 0.007(f'_c - 28) \geq 0.65 \quad (16)$$

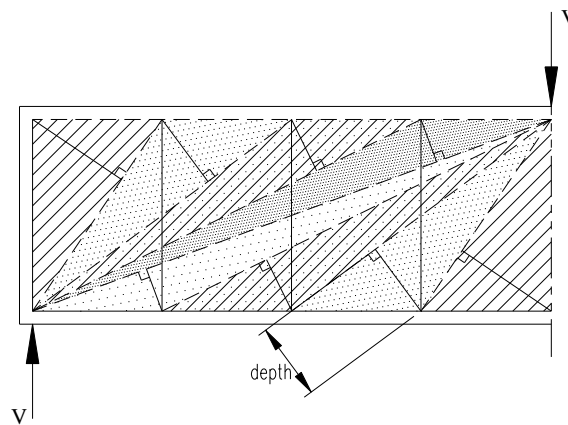


Fig. 3 Effective depths of inclined struts.

#### 4. VERIFICATION OF THE PROPOSED MODEL

##### 4.1. Experimental Database

To verify the accuracy of the proposed strut-and-tie model, the concrete beams reinforced with both longitudinal and transverse FRP bars failed by shear are collected. As discussed previously, the proposed model captures both beam and arch actions; therefore, only beams with an aspect ratio of from 2 to 3 displaying both beam and arch actions are collected to facilitate the verification of the proposed model. The collected beams were reinforced with various types of FRP reinforcements in both transverse and longitudinal directions, namely glass (GFRP), carbon (CFRP), aramid (AFRP) and basalt (BFRP). The concrete beams reinforced with FRP bars satisfied such requirements had been collected as shown in Table 1. The improved performance of the proposed model is identified by comparison with the available design code for structural concrete reinforced with FRP bars, namely the ACI 440.1R [3] and CSA S806 [2] design codes.



Table 1 Database of concrete beams reinforced with FRP bars.

ID	Beam				Long. Reinf.			Transverse Reinf.			
	$f'_c$ MPa	$b$ mm	$d$ mm	$a/d$	$A_f$ mm <sup>2</sup>	$E_f$ GPa	$f_{Fu}$ MPa	$A_{fv}$ mm <sup>2</sup>	$s$ mm	$E_{fv}$ GPa	$f_{Fwu}$ MPa
Nakamura & Higai [13]											
GG05-10	35.4	200	250	3.0	804	29	751	70	100	31	828
GG10-10	33.4	200	250	3.0	804	29	751	70	100	31	828
GG05-20	35.2	200	250	3.0	804	29	751	70	200	31	828
GG10-20	35.2	200	250	3.0	804	29	751	70	200	31	828
Zhao <i>et al.</i> [14]											
#10	34.3	150	250	3.0	1136	105	1124	57	90	39	1100
#16	34.3	150	250	3.0	852	105	1124	57	90	39	1100
#18	34.3	150	250	2.0	568	105	1124	57	90	39	1100
Maruyama & Zhao [15]											
#25	34.0	150	250	2.5	390	100	1200	77	120	30	600
#26	34.0	150	250	2.5	390	100	1200	77	120	30	600
#27	34.0	150	250	2.5	390	100	1200	77	60	30	600
#28	34.0	150	250	2.5	390	100	1200	77	60	30	600
#30	29.5	300	500	2.5	1560	100	1200	308	240	30	600
#32	29.5	450	750	2.5	3510	100	1200	692	360	30	600
Duranovic <i>et al.</i> [16]											
GB12	39.8	150	210	2.44	429	45	1000	40	153	45	1000
Alsayed [17]											
B2	35.7	200	320	2.28	794	42	764	63	80	43	656
Alkhrdaji <i>et al.</i> [18]											
BMI	24.1	178	279	2.69	1142	40	717	142	152	40	717
BMI	24.1	178	279	2.69	1142	40	717	142	203	40	717
BMV	25.2	178	279	2.69	1142	40	717	142	152	40	717
BMVI	25.2	178	279	2.69	593	40	717	142	203	40	717
Yang [19]											
GB52-P80	34.9	150	218	2.75	265	124	2068	60	150	27.9	720
Issa <i>et al.</i> [20]											
3-16B2	35.9	200	270	2.5	630	51	1060	164	127	53	1070

## 4.2. Discussion of Analytical Results

As shown in Table 2, the mean ratio of the experimental to predicted shear strength of concrete beams reinforced with FRP bars by the proposed strut-and-tie model and its standard deviation are 1.02 and 0.27, respectively. This showed that the proposed strut-and-tie model predicted accurately the shear strengths of concrete beams reinforced with FRP bars. The calculated shear strengths of concrete beams reinforced with FRP bars in the database following the available design codes are also showed in Table 2. The average ratios of the experimental to predicted strength and their standard deviation are 1.95 and 0.53; and 1.22 and 0.29 for ACI 440.1R [3] and CSA S806 [2], respectively. The comparison indicates that the proposed strut-and-tie model produces better statistical correlation than the ACI 440.1R [3] and CSA S806 [2] design codes for concrete beams reinforced with FRP bars with an aspect ratio of from 2 to 3.

The proposed model could provide a suitable tool to calculate the shear strength of concrete beams reinforced with FRP bars with an aspect ratio of from 2 to 3. The comparison also showed that the equations following the ACI 440.1R design code [3] are over-conservative in estimating the shear strengths of concrete beam reinforced with FRP bars.

Table 2 Experimental verification.

ID	Shear strengths				Experimental/Analytical Ratios		
	$V_{exp}$ kN	$V_{Pro}$ kN	$V_{CSA}$ kN	$V_{ACI}$ kN	$\frac{V_{exp}}{V_{Pro}}$	$\frac{V_{exp}}{V_{CSA}}$	$\frac{V_{exp}}{V_{ACI}}$
Nakamura & Higai [13]							
GG05-10	83	70	68	42	1.19	1.22	1.98
GG10-10	100	69.7	67	42	1.43	1.49	2.38
GG05-20	56	51.3	52	31	1.09	1.08	1.81
GG10-20	66	51.3	52	31	1.29	1.27	2.13
Zhao <i>et al.</i> [14]							
#10	113.7	124.4	108	58	0.91	1.05	1.96
#16	116.2	111.8	101	54	1.04	1.15	2.15
#18	123.3	131.1	106	50	0.94	1.16	2.47
Maruyama & Zhao [15]							
#25	110	78.9	65	40	1.39	1.69	2.75
#26	107	78.9	65	40	1.36	1.65	2.68
#27	131	109.9	82	59	1.19	1.60	2.22
#28	131	109.9	82	59	1.19	1.60	2.22
#30	370	325.2	256	157	1.14	1.45	2.36
#32	590	725.8	576	354	0.81	1.02	1.67
Duranovic <i>et al.</i> [16]							
GB12	67	48.5	42	24	1.38	1.60	2.79
Alsayed [17]							
B2	109	125	92	70	0.87	1.18	1.56
Alkhrdaji <i>et al.</i> [18]							
BMI	81.8	125.3	87	66	0.65	0.94	1.24
BMI	71.2	104.8	79	55	0.68	0.90	1.29
BMV	81	125.4	87	66	0.65	0.93	1.23
BMVI	52.9	92	64	49	0.58	0.83	1.08
Yang [19]							
GB52-P80	47.2	55.5	51.6	27.9	0.85	0.91	1.69
Issa <i>et al.</i> [20]							
3-16B2	134.7	168.8	145.5	98	0.80	0.93	1.37
Average					1.02	1.22	1.95
Standard deviation					0.27	0.29	0.53

## 5. PARAMETRIC STUDIES

The effects of various parameters on the shear strength of concrete beams reinforced with FRP bars are presented within this part of the paper through a parametric study. The parameters investigated are shear reinforcement ratios ( $\rho_{fw}$ ), longitudinal reinforcement ratios ( $\rho_f$ ), and aspect ratios (a/d).

### 5.1. The effects of shear reinforcement ratios ( $\rho_{fw}$ )

The analyse as shown in Fig. 4 was conducted to assess the effects of shear reinforcement ratios on the shear strength of concrete beams reinforced with FRP bars. The tests conducted by Nakamura and Higai [13] were chosen as reference specimens to assess these effects. Four levels of shear reinforcement ratios  $\rho_{fw}$  of 0.7%, 0.53%, 0.35%, and 0.18% were investigated for Nakamura and Higai’s test specimens [13].

Fig. 4 shows that with an increase in shear reinforcement content from 0.18% to 0.35%, 0.53%, and 0.7%; shear strength of beams increased by approximately 36.5%, 71.0% and 101.9%, respectively for Nakamura and Higai’s test specimens [13]. A similar trend was observed when comparing with the experimental results in Nakamura and Higai’s study [13]. The shear reinforcement ratios in Nakamura and Higai’s study [13] increased from 0.18% from 0.35%, the experimental results showed an increase of 48.2% as comparing with an enhancement of 36.5% following the proposed model.

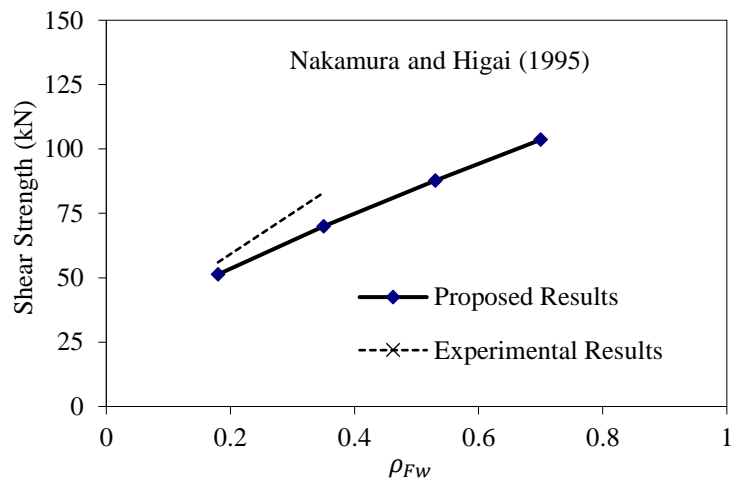


Fig. 4 Effects of shear reinforcement ratios.

### 5.2. The effects of longitudinal reinforcement ratios ( $\rho_f$ )

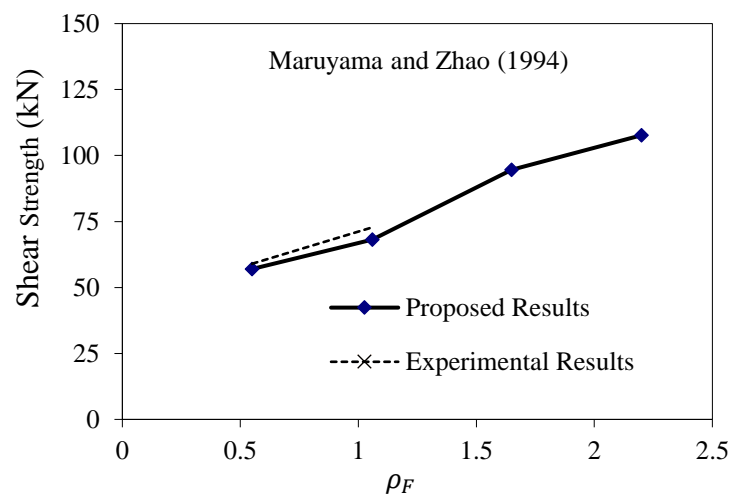


Fig. 5 Effects of longitudinal reinforcement ratios.

The effects of longitudinal reinforcement ratios on shear strengths of concrete beams

reinforced with FRP bars are presented in Fig. 5 for Maruyama and Zhao's tested specimens [15]. Four levels of longitudinal reinforcement ratios  $\rho_f$  of 2.20%, 1.65%, 1.06% and 0.55% were considered. As shown in Fig. 5, the shear strengths of Maruyama and Zhao's tested specimens [15] were observed to rise with an increase in longitudinal reinforcement ratios. The experimental results showed an increase of 23.4%; whereas, the analytical model illustrated a rise of 19.5% in the shear strengths of tested specimens when the longitudinal reinforcement ratio increases from 0.55% to 1.06%. This showed the significant influences of longitudinal reinforcement ratios on the shear strength of concrete beams reinforced with FRP bars.

## 6. CONCLUSIONS

This paper presents a strut-and-tie model to estimate the shear strength of concrete beams reinforced with FRP bars with an aspect ratio of from 2 to 3. The influences of shear reinforcement ratios ( $\rho_{fw}$ ) and longitudinal reinforcement ratios ( $\rho_f$ ) on the shear strength of concrete beams reinforced with FRP bars are investigated through a comprehensive parametric study based on the proposed method. Specific findings of this research are as follows:

Both beam and arch actions are considered in the proposed strut-and-tie model for concrete beams reinforced with FRP bars. The concrete contributions to the shear strengths are implemented into the model through the equivalent areas of FRP transverse reinforcements. The proposed model is verified with the available experimental results of concrete beams reinforced with various types of FRP bars. The average ratio of the experimental to predicted shear strength of concrete beams in the database by the analytical model is 1.02. This showed a good correlation between the analytical model and the experimental data.

The improved performance of the proposed model is identified by comparison with the available design codes for structural concrete reinforced with FRP bars. The comparison indicates that the proposed model produces better statistical correlation than the ACI 440.1R [3] and CSA S806 [2] design codes. The proposed model could provide a suitable tool to calculate the shear strength of concrete beams reinforced with FRP bars with an aspect ratio of from 2 to 3.

The effects of shear reinforcement ratios ( $\rho_{fw}$ ) and longitudinal reinforcement ratios ( $\rho_f$ ) on the shear strength of concrete beams reinforced with FRP bars had been investigated in the parametric study. And similar trends are observed as comparing between the experimental and analytical results.

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