Transport and Communications Science Journal, Vol. 74, Issue 1 (01/2023), 10-19

Transport and Communications Science Journal

MOMENT MODIFICATION FACTORS FOR THE BUCKLING DESIGN OF STEEL BEAMS – NEW RECOMMENDATIONS

Tien Nguyen Duy, Phe Van Pham*

University of Transport and Communications, No 3 Cau Giay Street, Hanoi, Vietnam

ARTICLE INFO

TYPE: Research Article
Received: 02/10/2022
Revised: 28/11/2022
Accepted: 05/01/2023
Published online: 15/01/2023
https://doi.org/10.47869/tcsj.74.1.2

* *Corresponding author*

Email: phe.phamvan@utc.edu.vn; Tel: +84865651184

Abstract. When a unbraced flexural steel beam is subjected to a ununiform moment distribution, a simplified moment modification factor (denoted as C_b) should be evaluated for the design of the buckling resistance of that member. However, typical standards for the buckling design of steel structures (e.g., American AISC A360, Australian AS-4100, Canadian CSA S16, Eurocode 3 and Japanese standards) currently recommend different design equations for the factor. Also, such equations are based on simplified expressions those are not exact solutions. Thus, the present study firstly revise the standard equations to discuss their advantages and disadvantages in application. Also, a numerical solution based on a finite element analysis package is then conducted in the present study to predict the C_b factor. The numerical solution is successfully validated against available research results. Based on the comparison of the modification factors between of the present numerical study and those based on the design standards, it is observed that the modification factors based on the current design standards maybe not safety enough to predict the buckling resistances in several loading cases. The present study finally recommends a new modification more on the safe side for the C_b factor to ensure a conservative design.

Keywords: buckling, flexural steel member, moment modification factor, standard, numerical solution, thin walled beams

1. INTRODUCTION

Steel beams often possess high resistances against tensioning, shearing, bending, and twisting loads. Thus, they are widely applied to build civil structures such as bridges and buildings. However, steel beams are often made in thin-walled forms. When not braced and subjected to bending loads, steel beams may have relatively complicated elastic lateral-torsional buckling responses. Many researchers and design specifications have paid a lot of attention to the design of the elastic buckling problem for the steel beams. Typical standards for the design of steel members (e.g, American AISC A360 [1], Canadian CSA S16 [2], Australian AS-4100 [3], Eurocode EC3 [4], Japanese JSCE [5]) recommended a simple procedure for the evaluation of the elastic buckling resistances in a given beam with arbitrary boundary and loading conditions. For a beam with a double symmetrical cross-section and with loads applied at the shear center, the procedure includes two steps in which the first step is to evaluate the elastic buckling resistance of a simply supported beam subjected to uniform bending moment (denoted as $M_{cr,uni}$), and the second step is to evaluate the elastic buckling resistance of the given beam (denoted as M_{cr}) which equals to a product of a moment modification factor (denoted as C_b) and the obtained $M_{cr,uni}$. Factor C_b depends on the moment distribution of the beam under different boundary and loading conditions. However, the typical standards proposed very different equations for the evaluation of the C_b factor. Based on a review, it is observed that Vietnamese standard TCVN 5575-2018 [6] does not even provide a factor of C_b for the design. Instead, it only provides basic solutions for the case in which loads are applied at the top or bottom flange centroids. Based on such scenarios, there have been several studies conducted to discuss the moment modification factors as well as to give warnings for design of steel structures. Hermanus [7] performed a comparative study of the moment modification factors between international steel design specifications. However, the study did not consider the Australian AS-4100 standard [3] and the Japanese JSCE standard [5]. Also, it did not consider the buckling response of fixed beams. Bresser et al. [8] developed a general formulation of equivalent moment factor for the elastic lateral torsional buckling of slender rectangular sections and I-sections. However, this study did not compare/discuss the difference of moment modification factors in the typical standards. The buckling of fixed beams was also not considered in their study. Manarin et al. [9] summarized the moment gradient factors for lateral-torsional buckling of T-shaped beams. Secer and Uzun [10] conducted a numerical investigation for the elastic lateral torsional buckling of simply supported beams under the combination of concentrated load and linear moment gradient. Again, the study did not consider the difference of the modification factor among various standards. Therefore, the present study is going to fill in the gap by performing a review on the typical standard equations for the moment modification factors of simply supported beams and fixed beams subjected to an arbitrary point load. Also, a detail comparison between the equations will be discussed and key warnings will be made. A numerical study will be conducted to have a further view on the modification factor. Finally, a new recommendation will be proposed to improve the standard equations for the moment modification factor.

2. DESCRIPTION OF THE PROBLEM

The present study is going to investigate the effect of internal moment distributions on the elastic lateral-torsional buckling load of a steel beams. A simply supported beam (Fig. 1a) and a fixed beam (Fig. 1b) with a double symmetric cross-section and subjected to a point load P applied at the sectional shear center are considered. The point load is applied a distance of x from the left support. It is required to evaluate the moment buckling resistance, to compare the moment modification factors of such systems under different distance of x based on 5 typical standards for the design of steel structures [1-5] and based on a the present numerical solution, so as to introduce a new recommendation for the moment modification factor.

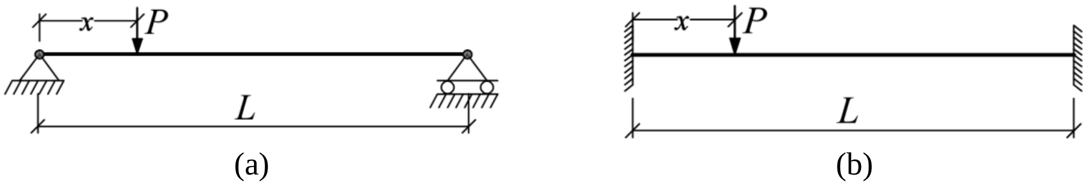


Figure 1. Beams under a point load P (a) Simply supported ends, (b) Fixed at both ends.

3. MOMENT MODIFICATION FACTOR BASED ON STANDARDS [1-5]

The first step for the evaluation of the elastic lateral-torsional buckling resistance based on the design standards [1-5] for a given thin walled steel beam with arbitrary boundary and loading conditions is to evaluate the buckling resistance $M_{cr,uni}$ of a simply supported beam under the application of a uniform moment (Fig.2 and Eq. (1)).

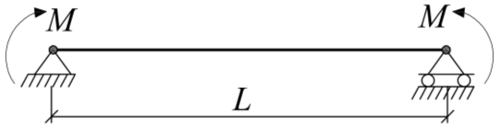


Figure 2. Simply support beam under a uniform moment.

$$M_{cr,uni} = \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \quad (1)$$

in which I_y is the moment of inertial about the weak axis, J is the St.Venant torsional constant, C_w is the warping constant, E is the elastic modulus, G is the shear modulus of the simply supported beam under uniform moment.

The second step is to evaluate the buckling resistance M_{cr} of the given beam. Because the given beam may has a non-uniform bending moment diagram, typical standards of steel structures [1-5] propose moment modification factors C_b and they recommend to relate the buckling resistance M_{cr} with the resistance $M_{cr,uni}$ through C_b as follows

$$M_{cr} = C_b M_{cr,uni} \quad (2)$$

The following sub-sections are going to revise the C_b factors proposed in the typical standards for steel member designs [1-5].

3.1. Moment modification factor based on American standard

Kirby and Nethercot [11] proposed a simplified equation for the evaluation of the moment modification factor C_b. The equation can be applied for beams with single span, with double symmetric cross-section. The equation is currently presented in Chapter F of AISC A360-10 (Equation C-F1-2) [1] and it is recalled in Eq. (3) in the following.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \quad (3)$$

where $M_{\max}$ is the absolute value of maximum moment in the unbraced segment, M_A is the absolute value of moment at quarter point of the unbraced segment, M_B is the absolute value of moment at centerline of the unbraced segment, M_C is the absolute value of moment at three-quarter point of the unbraced segment.

3.2. Moment modification factor based on Australian standard

The Australian AS-4100 standard [3] provides a general simplified expression for the beams that are laterally bending and warping free as presented in Eq. (4) in the following

$$C_b = \frac{1.7M_{\max}}{\sqrt{M_A^2 + M_B^2 + M_C^2}} \tag{4}$$

where $M_{\max}$, M_A, M_B, M_C are determined in a similar manner as presented in Eq. (3).

3.3. Moment modification factor based on Canadian standard

The Canadian CSA S16 standard [2] provides the following simplified solution for C_b

$$C_b = \frac{4M_{\max}}{\sqrt{M_{\max}^2 + 4M_A^2 + 7M_B^2 + 4M_C^2}} \le 2.5 \tag{5}$$

where $M_{\max}$, M_A, M_B, and M_C are similar to those presented in Eq. (3).

3.4. Moment modification factor based on Eurocode 3 and Japanese standards

Eurocode 3 and Japanese standards [4,5] don’t provide a simplified equation for the determination of the moment modification factor. Instead, Eurocode 3 gives several values for different loading cases in its Table 6.6 of Clause 6.3.2. For a simply supported beam subjected to a midspan point load P, the modification factor is given as 1.365. For a fixed beam subjected to a midspan point load P, the factor is given as 1.565. Also, Japanese standard [5] only provides 2 factors for two cases of midspan loading in its Table C4.3.6 of Section 5.3.3. For a simply supported beam subjected to a midspan point load P, the modification factor is given as 1.365. For a fixed beam subjected to a midspan point load P, the factor is given as 1.736 (Table 1). Both standards [4,5] don’t provide the factors for other positions of the point load P.

Table 1. Summarization of the moment modification factors based on Eurocode 3 [4] and Japanese standard [5] when load P is applied at midspan.

Standard	Simply supported beams	Fixed beams
Eurocode 3 [4]	1.365	1.565
Japanese standard [5]	1.365	1.736

3.5. Evaluations of the moment modification factors based on Standards [1-5]

Based on Figs. 1a,b, by setting x at points distanced at $L/16$, by plotting internal moment diagrams, and by evaluating values of $M_{\max}$, M_A, M_B, M_C for Eqs. (3,4,5), one obtains the values of C_b for the simply supported and fixed beams based on the American AISC standard [1] (denoted as AISC), those based on the Australian AS-4100 standard [2] (denoted as AUS), those based on the Canadian CSA S16 standard [3] (denoted as CAN), as summarized in

Table 2a,b. The modification factor based on Eurocode 3 [4] is denoted as EC3, while that based on Japanese standard [5] is denoted as JAP (Table 2a,b). It can be observed from Table 2 that C_b factors based on the standards are relatively different to each other. For example at distance $x/L = 0.375$ of the simply supported beam (Table 2a), the C_b factor based on the AISC is 1.404, while that based on the AUS is 1.524, corresponding to a difference of 8.6%.

Table 2. Evaluation of the moment modification factors based on typical standards.

x/L	AISC	EC3	AUS	CAN	JAP
0.0625	1.596		1.704	1.656	
0.125	1.522		1.590	1.563	
0.1875	1.444		1.477	1.467	Not given
0.25	1.364	Not given	1.363	1.368	
0.3125	1.404		1.490	1.433	
0.375	1.404		1.524	1.423	
0.4375	1.373		1.483	1.362	
0.5	1.316	1.365	1.388	1.265	1.365
0.5625	1.373		1.483	1.362	
0.625	1.404		1.524	1.423	
0.6875	1.404		1.490	1.433	
0.75	1.364	Not given	1.363	1.368	Not given
0.8125	1.444		1.477	1.467	
0.875	1.522		1.590	1.563	
0.9375	1.596		1.704	1.656	

(a) Simply supported beams

x/L	AISC	EC3	AUS	CAN	JAP
0.0625	2.052		2.238	2.123	
0.125	1.923		1.999	1.949	
0.1875	1.786		1.768	1.766	
0.25	1.642	Not given	1.546	1.577	Not given
0.3125	1.968		2.357	2.020	
0.375	2.095		2.492	1.978	
0.4375	2.065		2.136	1.720	
0.5	1.923	1.565	1.700	1.414	1.736
0.5625	2.065		2.136	1.720	
0.625	2.095		2.492	1.978	
0.6875	1.968		2.357	2.020	
0.75	1.642	Not given	1.546	1.577	Not given
0.8125	1.786		1.768	1.766	
0.875	1.923		1.999	1.949	
0.9375	2.052		2.238	2.123	

(b) Fixed beams

4. MOMENT MODIFICATION FACTOR BASED ON A NUMERICAL SOLUTION

Because the moment modification factors based on the typical standards [1-5] are relatively different to each other as discussed above, a numerical study is conducted to evaluate the moment modification factors so as to have better discussions for the factor. The numerical solution is based on ABAQUS software. A 5m-span steel beam with a cross-section W250x45 is considered. The steel beam model is created in very similar way as done in [12], i.e., the beam is meshed by using C3D8R elements through 5 independent numbers of elements n_1 to n_5 (Fig. 3). A mesh sensitivity is conducted and the mesh, with it the convergence of the moment resistances are obtained, includes $n_1 = 20$, $n_2 = n_3 = 4$, $n_4 = 40$, $n_5 = 400$ elements.

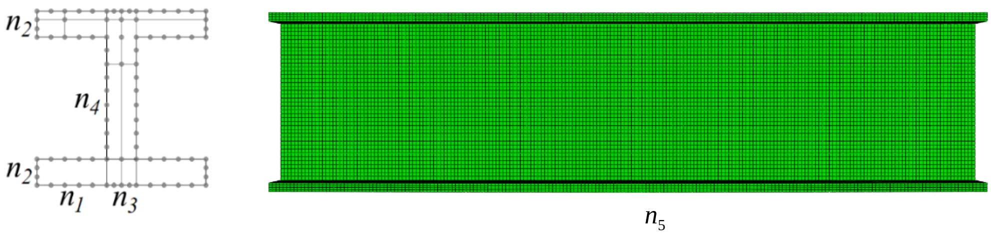
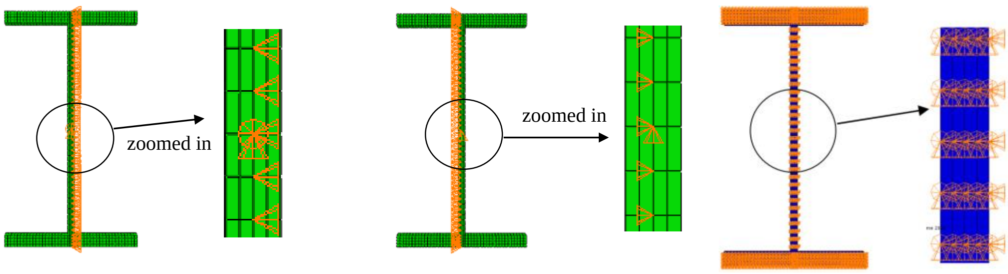


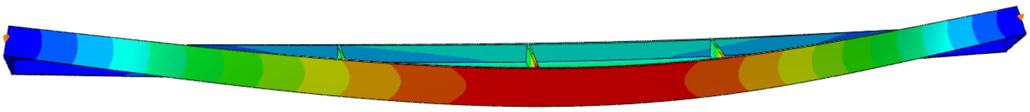
Figure 3. Independent numbers of elements controling the mesh of the beam.

Boundary conditions of simply supports are modelled as presented in Figs. 4a,b, while those of the fixed supports are presented in Fig. 4c. The beam is subjected to the load as presented in Fig. 2 and Figs. 1a,b. The buckling resistances $M_{cr,uni}$ and M_{cr} can be obtained from the finite element analyses. Factor C_b of the numerical study can be then obtained by using Eq. (2). The buckling configurations of a simply supported beam and a fixed beam subjected to a midspan point load P in the present numerical study are depicted in Figs. 5a,b.

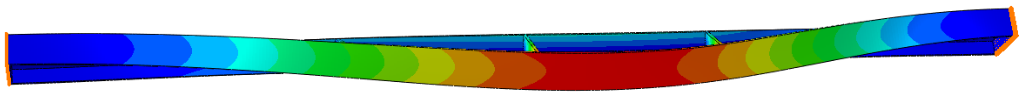


(a) Pin support (b) Roller support (c) Fixed support

Figure 4. Boundary condition of the problem.



(a)



(b)

Figure 5. Buckling configuration of (a) simply supported beam and (b) fixed beam when load P is applied at the midspan.

Validation of the numerical study: Based on the present numerical solution, $M_{cr,uni}$ can be evaluated as $123\,kN.m$. Meanwhile, M_{cr} for $x/L = 0.5$ of the simply supported beam is 166.6 *kN.m*, corresponding to a $C_{b,num}$ factor of 1.355. Also, M_{cr} for $x/L = 0.25$ of the simply supported beam is 179.1 *kN.m*, corresponding to a $C_{b,num}$ factor of 1.456. Based on the well-known study of Nethercot and Trahair [13], C_b factor for the case of $x/L = 0.5$ is 1.353 and that for the case of $x/L = 0.25$ is 1.445. By comparing the C_b factors obtained from the present numerical solution and the previous study [13], the difference for the case of $x/L = 0.5$ is only 0.1% and that for the case of $x/L = 0.25$ is 0.8% (Table 3). The small differences indicate that the present numerical solution can excellently capture the buckling resistances of the given beams.

Table 3. Comparison of Cb between the present 3D FEA solution against Nethercot and Trahair [13].

x/L	Present 3D FEA solution	Nethercot and Trahair [13]	% difference
(1)	(2)	(3)	(4) = (3-2)*100%/(2)
0.5	1.355	1.353	0.1
0.25	1.456	1.445	0.8

5. DISCUSSIONS AND RECOMMENDATIONS FOR THE MOMENT MODIFICATION FACTORS

Figure 6 presents the relationship between the moment modification factor C_b and the non-dimensional load position ratio x/L for the simply supported beam, as obtained from AISC, CAN, AUS, EC3 and JAP standards (Section 3.5 above) and from the present numerical solution (Section 4 above). It is observed that (***i***) the C_b curves based on AISC, CAN and AUS standards are divided into 4 segments, in which the first and the forth segments (i.e., $0.0625 \le x/L \le 0.25$; $0.25 \le x/L \le 0.9375$) are almost linear, while the second and the third segments (i.e., $0.25 \le x/L \le 0.5; 0.5 \le x/L \le 0.75$) are concave down. Meanwhile, the present numerical solution predicts a continuous curve for the C_b factor and the curve is concave up. It is noticed that the present solution is consistent with the research results reported by the thesis of Hermanus [7], and it is believed that there maybe no reason that the standard C_b curves at the third and the fourth segments are concave down. It is recalled that factors C_b of the AISC, CAN, AUS standards are based on simplified equations (i.e., Eqs. (3), (4), (5)) and thus the concave-down segments may not reflect the correct C_b factor.

From Fig. 6, it is also observed that (***ii***) when the load is applied at $x/L = 0.375$, the C_b factor based on the present numerical solution is 1.378, while that based on the AISC standard is 1.404, that based on the CAN standard is 1.423 and that based on the AUS standard is 1.524. Therefore, the C_b factor based on the present numerical study is the smallest value, while that based on the AUS standard is the highest value, among the four solutions. This suggests that the C_b factors based on the AISC, CAN, AUS standards may not be conservative enough when load is applied at the middle of the second and third segments (e.g.., $x/L = 0.375$).

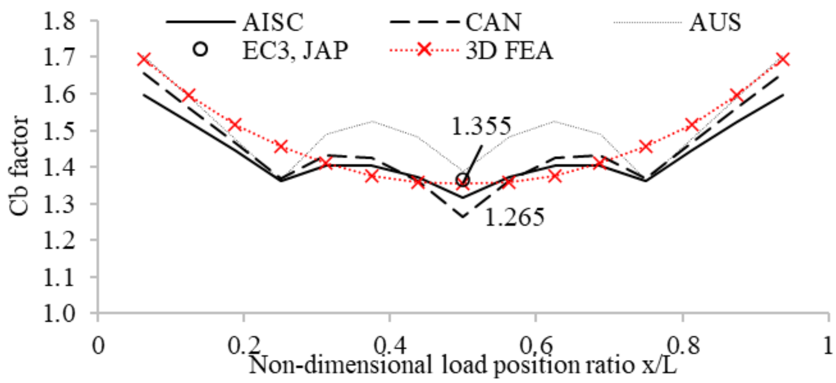


Figure 6. Relationship between C_b and x/L for the given simply supported beam.

Figure 7 presents the relationship between the moment modification factor C_b with the non-dimensional load position ratio x/L as obtained from AISC, CAN, AUS, EC3 and JAP standards (as presented in Section 3.5) and from the present numerical solution (as presented in Section 4) for the fixed beam. It is again observed that the C_b factors based on the numerical study is a continuous concave-up curve, while those based on the AISC, CAN, AUS standards include four segments in which the second and third segments are concave down. The C_b factors based on the numerical study are relatively higher than those of the AISC, CAN solutions. Also, the solutions based on the AISC and CAN standards are much more conservative than that based on the AUS standard. However, factor C_b at $x/L = 0.375$ of the numerical solution is 2.400, while that of the AUS standard is 2.492. This means that the AUS standard may not be conservative enough for prediction of the C_b factor when the point load is applied at $x/L = 0.375$.

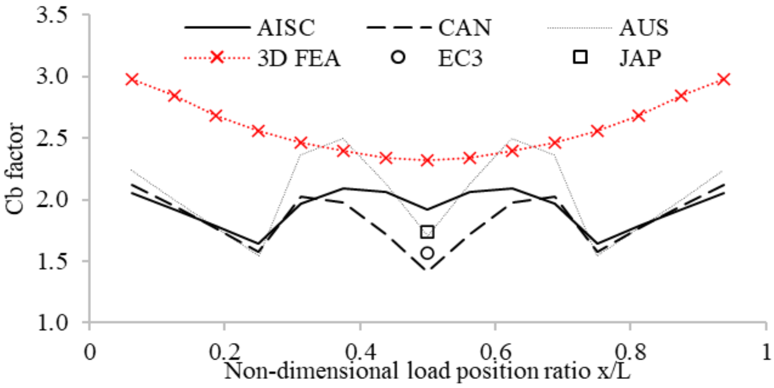


Figure 7. Relationship between C_b and x/L for the given fixed beam.

Based on the observations for the simply supported beams in Fig. 6 above, it is found that the CAN standard provides a lowest value (i.e., a conservative solution) for the moment modification factors C_b when the point load is applied at $x/L = 0.5$. Also, the C_b curve is conservative for the simply supported beam, exempt the case in which load is applied around $x/L = 0.375$ or $x/L = 0.625$. Hence, the present study recommends a small change for the current CAN - C_b curve. As discussed, the current CAN - C_b curve includes four segments (i.e., the first one is almost linear for $x/L \le 0.25$, the second one is concave down from $0.25 \le x/L \le 0.5$, the third one is concave down from $0.5 \le x/L \le 0.75$, and the forth one is almost linear for $0.75 \le x/L$). The recommendation of the present study is to make a linearization for the second and the third segments while keeping the first segment, the forth segment and the C_b value at $x/L = 0.5$ unchanged, as depicted in Fig. 8. This ensures a conservative solution for the C_b factor.

For the fixed beams, it is recommend to evaluate the C_b factor based on CAN or AISC standards because they are more conservative than other standards and the present numerical solutions, as discussed in Fig. 7.

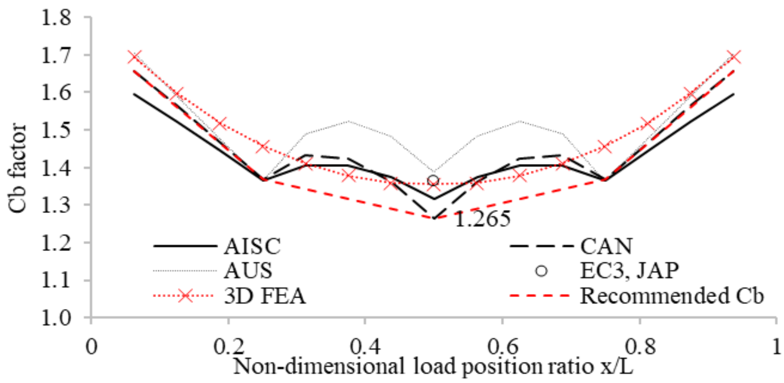


Figure 8. Recommendation of the Cb curve for the simply supported beams.

6. CONCLUSION

The present study has performed a detailed review on the moment modification factors C_b of simply supported beams and fixed beams subjected to an arbitrary point load applied at the section shear center. Five typical standards for the design of steel structures were considered and they included the American AISC, Australian AS-4100, Canadian CSA-S16, Eurocode 3, and Japanese JSCE standards [1-5]. Also, a numerical model implemented in a commercial software was successfully developed in the present study to predict the C_b factors. Comparisons and discussions of the C_b factors based on the standards and the numerical solution were then presented. Key conclusions of the present study are summarized in the following:

(i) The C_b curves based on the AISC, CAN and AUS standards against the position x/L of a point load are divided into 4 segments, in which the first and the forth segments (i.e., x/L $\leq$0.25 or 0.75 $\leq$ x/L) are almost linear, while the second and the third segments (i.e., 0.25 $\leq$ x/L $\leq$0.5 or 0.5 $\leq$ x/L $\leq$0.75) are concave down. Meanwhile, the present numerical solution predicts a concave-up continuous C_b curve. It is believed that there maybe no reason that the C_b curves at the third and the fourth segments of the AISC, CAN and AUS standards are concave down and that the concave-down segments of the AISC, CAN, AUS standards may not reflect the correct value of C_b factor.

(ii) When the point load is applied at x/L=0.375 of a simply supported beam, the C_b factor based on the present numerical study was smaller than those of the AISC, AUS, CAN standards. This suggested that the C_b factors based on the AISC, CAN, AUS standards maybe not conservative enough for the case.

(iii) The present study recommends a small modification for the C_b factor of simply supported beams based on the C_b curve of the current CAN standard. The modification was to make a linearization for the second and the third segments while keeping the first segment, the forth segment and the C_b value at x/L=0.5 unchanged, so as to ensure a conservative solution for the C_b curve.

(iv) The present study recommends to evaluate the C_b factors based on CAN or AISC standards for the fixed beams, because they are conservative enough.

ACKNOWLEDGMENT

This research is funded by University of Transport and Communications (UTC) under grant number T2022-CT-007TD

REFERENCES

[1]. American Institute of Steel Construction (AISC). Specification for Structural Steel Buildings, ANSI/AISC 360-10,. Chicago, IL: AISC, 2010. https://www.aisc.org/Specification-for-Structural-Steel-Buildings-ANSIAISC-360-10-2010

[2]. CSA S16, Limit states design of steel structures, Standard CAN/CSA-S16-14, Canadian Standards Association, Mississauga, Ontario, 2014. https://www.csagroup.org/store/product/S16-14/

[3]. Standards Australia. AS-4100 1998. AS-4100 Steel Structures, Sydney, Australia, 1998 https://store.standards.org.au/product/as-4100-1998

[4]. EN 1993-1-1:2005 (E): Eurocode 3: design of steel structures—part 1–1: general rules and rules for buildings, CEN, 2005. https://www.phd.eng.br/wp-content/uploads/2015/12/en.1993.1.1.2005.pdf

[5]. Japan Society of Civil Engineers (JSCE) - Standard Specifications for Steel and Composite Structures, 2009. https://www.jsce-int.org/system/files/Standard.pdf
[6]. TCVN 5575-2018, Steel structures – Design standard, Vietnam institute for Building Science and Technology (INST), 2018. https://tieuchuan.vsqi.gov.vn/tieuchuan/view?sohieu=TCVN+5575%3A2012
[7]. J. W. S. Hermanus, Comparative study of the equivalent moment factor between international steel design specifications, Master thesis, University of Stellenbosch, 2014. https://scholar.sun.ac.za/handle/10019.1/95863
[8]. D. Bresser, G. J. P. Ravenshorst, P. C. J. Hoogenboom, General formulation of equivalent moment factor for elastic lateral torsional bucklingof slender rectangular sections and I-sections, Engineering Structures, 207 (2020) 110230. https://doi.org/10.1016/j.engstruct.2020.110230
[9]. M. Manarin, R. G. Driver, Y. Li, Moment gradient factor for lateral torsional buckling of T-shaped beams, Proceeding of the Annual Stability Conference, Missouri, 2019. https://www.aisc.org/globalassets/continuing-education/ssrc proceedings/2019/manarin_et_al_ssrc_2019.pdf
[10]. M. Secer, E. T. Uzun, Elastic lateral torsional buckling of simply supported beams under the combination of concentrated load and linear moment gradient, IOP Conf. series: Materials Science and Engineering, 245, 2017.https://iopscience.iop.org/article/10.1088/1757-899X/245/3/032077
[11]. P. A. Kirby, D. A. Nethercot, Design for Structural Stability, John Wiley Sons, Australia, 1979. https://www.amazon.com/Design-Structural-Stability-P-Kirby/dp/0003830462
[12]. H. Cao, D. T. Nguyen, V. P. Pham, T. T. Bui, D. B. Nguyen, Comparison of inelastic moment resistances of rolled steel beams based on different specifications and a numerical study, Transport and Communications science journal, 73(2022) 16-30. https://doi.org/10.47869/tcsj.73.1.2
[13]. D. A. Nethercot, N. S. Trahair, Lateral Buckling Approximations for Elastic Beams, The Structural Engineer, 54 (1976) 197-204. https://trid.trb.org/view/66407