



## OPTIMAL SUPERCAPACITOR PLACEMENT IN AN URBAN RAILWAY LINE

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**Abstract.** Supercapacitors (SCs) are important devices used in renewable energy storage applications on urban railways due to their high power density, good performance, and long maintenance-free lifetime. The position and capacity of supercapacitors play an essential role in increasing energy efficiency and improving the operating condition of the power supply system. This paper proposes an optimal methodology to place a supercapacitor energy storage system (SESS) for urban railway lines. The proposed method uses simulation tools to determine the level of renewable energy as well as the cycle of renewable energy occurrence at substations. Next, based on the working characteristics of the supercapacitor to calculate the accumulated energy that can be reused. Finally, the problem of the optimal siting and sizing of the SESSs is solved to maximize the economic benefits. A case study is applied to evaluate the algorithm. The simulation results demonstrate that installing the optimal SESS can increase energy efficiency, lower transient power, and the solution found is the best choice for economic goals.

**Keywords:** optimization, supercapacitor, urban railway, energy storage system, energy efficiency, regenerative braking

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### 1. INTRODUCTION

According to the energy consumption of urban railways for different mode operations, renewable energy accounts for about 53% of the total consumed energy to be released during the transition period. By subtracting the loss of drag and mechanical movement, about 22%-40% of the energy consumed can be recovered by electrical regeneration [1]. In this sector,

researchers are focusing on the development of new solutions and techniques to improve the efficiency and systems' capacity to achieve higher energy savings [2]. One of the possible strategies is the recovery of the energy produced by the trains during the braking phases [3]. Installing the energy storage systems (ESSs) which store braking energy surplus and return it to the contact line when necessary, is an effective solution for implementing this strategy. Depending on where they are installed, there are both mobile ESSs (located onboard vehicles) and stationary ESSs (installed in electrical substations or along the track). Compared to the case of onboard ESSs, there is no restriction of installation space and weight, but the line transmission losses are higher. Thus, attention must be paid to the position of the storage equipment within the trackside [4].

An efficient energy storage system not only reduces the energy consumption but also stabilizes the line voltage and reduces the peak input power, resulting in lower losses in the electric lines. The urban trains generate high instantaneous currents when they are braking. The braking time is around 10 to 15 seconds, therefore the amount of regenerative power is very high and it is hard to find an appropriate ESS that can store these high currents in such a few periods. Fortunately, super-capacitors have special features such as long life, rapid charging, low internal resistance, high power density, and a simple charging method. These features make super-capacitors suitable for the recovery of the regenerative braking energy in a metro network [5].

Regarding the stationary ESSs implementation, the problem of determining the location and capacity of supercapacitor energy storage systems along the tracks is solved as an economic-technical optimization solution. Many solutions have been proposed. In [6] the authors provided a solution based on the analysis of regenerative currents at stations to calculate recoverable energy in a year. From there, determine the capacity of the supercapacitor suitable for the regenerative current and capable of effectively recovering the regenerative energy. To obtain the best energy savings and voltage profile by optimizing the location and size of ultra-capacitors, Wang [7] applied a methodology for optimal ultra-capacitor energy storage system locating and sizing is put forward based on the improved genetic algorithm. The given method was evaluated in the example of a Chinese metro line. The obtained result is an optimized scheme of full days with an average energy saving rate of 4.88%, a regeneration cancellation rate of 5.45%, and an installation cost of 3.50 M\$. In another study, Ratniyomchai [8] focused on a criterion to identify the most suitable capacities and locations of the storage devices to minimize the electrical energy consumption of the train and total line losses. The results confirm that minimal capacitances are obtained when supercapacitors are close to the substations for each section of the electrified line.

Further, a novel optimization method that combines genetic algorithms and a simulation platform of urban rail power supply system is proposed by Xia [9], which can obtain the best energy management strategy, location, and size for ESSs simultaneously. This study aims to optimize the energy management, location, and size of stationary super-capacitor ESSs simultaneously and obtain the best economic efficiency and voltage profile of metro systems. Investigating the design of stationary ESSs based on supercapacitors for metro networks,

Calderaro [5, 10] proposed a heuristic method for the joint siting and sizing design of stationary supercapacitors. In these studies, a new formulation of the ESSs siting and sizing optimization problem was given and solved using the particle swarm algorithm.

Energy management strategy and configuration for ESSs will influence each other, and they both affect the performances of the urban railway network. So if considering the power flow to be fixed, it will cause errors in the calculation. Recently, new methods and new formulations to improve the accuracy and efficiency in solving the problem of determining the optimal installation configuration of supercapacitors have been researched and proposed. Lamedica [11] developed software using the PSO algorithm, whose objective function is based on the net present value. David [12] proposed a model in which nature-inspired optimization algorithms are applied in combination with a very realistic railway simulator.

This paper also focuses on solving the problem of finding the optimal location and capacity of supercapacitor energy storage systems (SESSs) for urban railway lines. The objective function is to minimize the total cost of energy consumption and installation costs. Because the two parameters of location and capacity of SESSs both greatly affect the energy distribution throughout the railway power supply system. Therefore, the conception of the proposed method is to fix one parameter then finding the other parameter. In this paper, the location of SESSs is limited to installation at substations only. Then the space of the location variable is not large, and this parameter will be chosen as a fixed one. The proposed method is divided into two tasks: first is to determine the optimal location to install SESS; the second is to determine the optimal capacity of SESSs to achieve the minimum total cost. The second task will be solved first by determining the optimal capacity of the corresponding SESSs for each SESS installation configuration. From there going back to the first task, compare the cost function of all the configurations to find the optimal configuration with the lowest cost function. The obtained configuration indicates where to install the SESSs. A case study is used to evaluate the feasibility and effectiveness of the proposed method.

## **2. MODELLING THE POWER SYSTEM OF AN URBAN RAILWAY LINE**

The DC power supply system includes substations, contact lines, trainloads, tracks, and return lines. The traction current starts from the substation (rectified) leads on the contact network to the trains. Through the running track and then the return lines, the current will return to the substation. From the point of view of the electrical circuit model, the substation is described by a DC supply; trains are described by a variable value current source, and the contact line and tracks are described by distributed resistors.

The model of a typical urban train network with stationary ESSs is shown in Figure 1.

Trains are modeled as current sources absorbing power at the accelerating time and generating power at the regenerative braking time. For a given speed, the power at the wheels is calculated. This power is needed to overcome vehicle inertia, slopes and curves, aerodynamic friction, and rolling friction. Going upstream the vehicle components and their related efficiencies, the power requested from the contact line is determined as follows:

$$P_{tr}(t) = \begin{cases} \frac{F_k(t) * v(t)}{\eta_k} + P_{phu} \\ -F_h(t) * v(t) * \eta_h + P_{phu} \end{cases} \quad (1)$$

where:

$v(t)$ - train speed, is determined according to the speed profile.

$F_k(t)$  – traction force, that is determined relative to the operating speed from the speed-traction force characteristic.

$F_h(t)$  – braking force, which is determined relative to the operating speed from the speed-braking force characteristic.

$\eta_k$  – efficiency in traction mode, which includes the gearbox efficiency, the motor efficiency, and the inverter efficiency.

$\eta_h$  – efficiency in braking mode.

$P_{phu}$  – the power required for accessories services.

Substations are simply modelled as ideal DC voltage sources with series resistance.

Supercapacitors are connected to the traction system utilizing a bi-directional DC-DC converter. The supercapacitor voltage and current are  $v_{sc}(t)$  and  $i_{sc}(t)$  respectively. The energy of the supercapacitor module can be calculated as:

$$E_{sc}(t) = \int_0^t P_{sc}(t) dt = \int_0^t v_{sc}(t) * i_{sc}(t) dt = \frac{1}{2} C_{sc} \int_0^t v_{sc}^2(t) dt \quad (2)$$

The supercapacitor served as a constant power load in DC power flow. First, based on the amount of voltage of the network and the working state of the supercapacitor, it is decided that the supercapacitor should be in either charging, discharging, or standby mode. Then, by using the previous iteration voltage, the current of the supercapacitor is calculated, DC network power flow is performed and the new amount of supercapacitor voltage can be determined. At each simulation time step  $t(i+1)$ , the amount of energy of ESS,  $E_{sc}(i+1)$ , is derived from (3):

$$\begin{aligned} E_{sc}(i+1) &= E_{sc}(i) + P_{sc}(i) * \Delta t \\ \Delta t &= t(i+1) - t(i) \end{aligned} \quad (3)$$

The contact lines and tracks are modelled as electric resistances. Since trains are moving between subsequent stations or substations, the electrical resistance between the train and the initial substation and the electrical resistance between the train and the next substation, are time-variant. Therefore, at each time point, the resistance of one section between two trains and between the train and the substation is calculated as follow:

$$\begin{aligned} R_{1-2} &= R_0 * |x_1 - x_2| \\ R_{sst-2} &= R_{ng} + R_0 * |x_{sst} - x_2| \end{aligned} \quad (4)$$

where  $R_0$  is the resistance of contact line and track per one km; then  $x_1$  và  $x_2$  is the location of two subsequent trains;  $x_{sst}$  is the location of substation;  $R_{ng}$  is the resistance of

feeder.

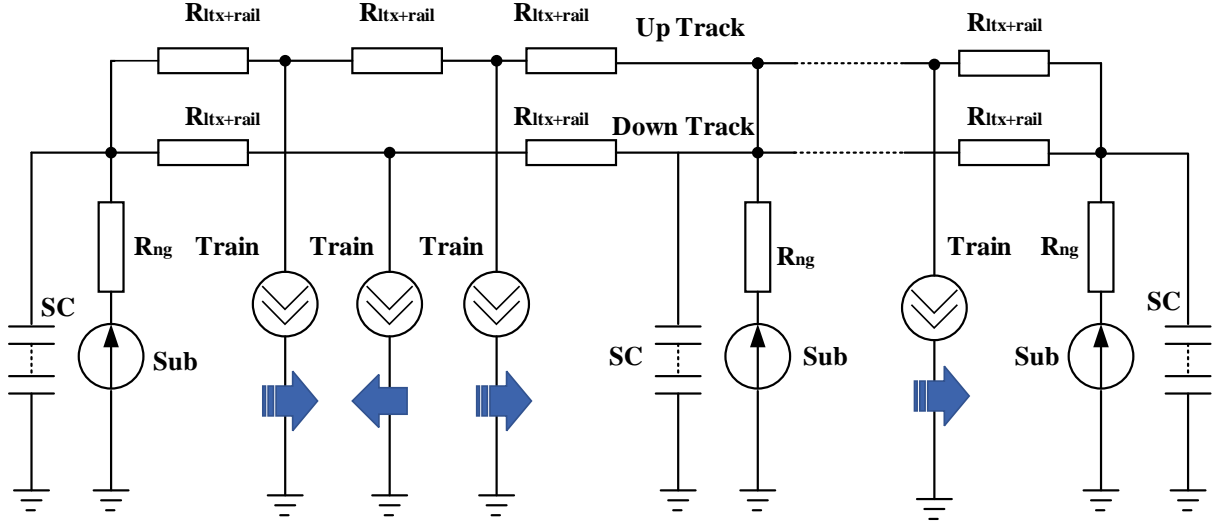


Figure 1. Urban railway line model with the supercapacitor storage system.

### 3. OPTIMIZATION METHOD

#### 3.1. Problem formulation

The main objective of this research is to determine the location and capacity of supercapacitors to achieve the maximum benefit. Therefore, the problem is defined as the minimization of the total cost of consumption energy for operating the railway system and the cost relatives to the investigation as well as maintenance of supercapacitors, which is shown in (5):

$$\min \sum_{k=1}^{n_{sub}} (energy\_cost(k) + sc\_cost(k)) \quad (5)$$

*subject to:*  $0 \leq E_{sci} \leq E_{max}$

In there,

$energy\_cost(k)$  is the annual energy cost of substation  $k$  obtained with the supercapacitor capacity  $E_{sci}$ .

$$energy\_cost(k) = E_{sub\_k}(E_{sci}) * 365 * ny * cost_{elec}$$

$sc\_cost(k)$  is the cost for investigation and maintenance of the supercapacitor energy storage system.

$$sc\_cost(k) = cost_{instal} + \frac{E_{sci}}{T_{charging}} * (cost_{sc} + cost_{conv} + cost_{main} * 365 * ny)$$

$E_{sub\_k}$ : consumption energy of substation  $k$  per day with the capacity of the supercapacitor is  $E_{sci}$ .

$E_{sci}$  presents the capacity of the supercapacitor with the position at substation  $k$ . The values for this variable are going from 0 kW (no supercapacitor installed at substation  $k$ ) in steps of  $\Delta E_{sc} = 1kW$  to the  $E_{max}$ .  $E_{sci}$  is calculated according to equation (2).

$cost_{elec}$  is the price of one kWh of electricity.

$ny$  presents the number of years to estimate energy efficiency.

$T_{charging}$  is the maximum energy charging cycle of the supercapacitor in one operating cycle.

$cost_{instal}$  is the cost for installing a supercapacitor storage system.

$cost_{sc}$  is the cost of one kW supercapacitor.

$cost_{conv}$  is the cost of one kW DC/DC converter.

$cost_{main}$  is the maintenance and replacement cost of the supercapacitor storage system per one day.

### 3.2. Optimal placement algorithm of supercapacitors for a railway line

The energy cost of the system and the installation cost of a supercapacitor energy storage system both depend on the location and capacity of the SESSs. For a particular SESS location, when the capacity value of SESS is different, the ability to recover regenerative energy is different, thereby determining the energy distribution throughout the traction power supply network accordingly. For a particular energy distribution, it is possible to calculate the value of the energy cost function. In this paper, the energy distribution is determined by the power flow algorithm. The positions of the SESSs are fixed into  $2^{n_{sub}}$  configuration unions corresponding to  $n_{sub}$  substations. For each configuration union, we will find the optimal capacity of SESSs so that the cost function reaches the minimum value. Among the configuration unions, find the configuration with the smallest cost function that is the optimal result.

The procedure of the proposed algorithm is presented as follows:

Step 1. Calculating the power of the train.

This step calculates the power of one train according to the time when moving along the railway line from the initial station to the final station in one direction. This power is specified according to equation (1).

Step 2. Setup the initial parameters.

Each operating scenario will determine the specific power mode on the railway power network. In this step, an average interval of time between the trains is set up for a specific scenario. Besides, the first union of supercapacitor location is also initialized.

Step 3. Check whether all union has been tested or not?

This step checks the stop condition of the tested procedure. If satisfied, go to step 7, otherwise move to step 4.

Step 4. Determining the power of substations in a period of headway.

In this step, firstly it is necessary to form the electric circuit of the DC power network at each point in the headway duration. Subsequently, the power value estimation at substations is carried out using the power flow algorithm. Finally, the power of substations during the headway period is determined by combining the values at each time point above.

Step 5. Determine the optimal capacity of supercapacitors installed at each substation.

Based on the recoverable regenerative energy determined in the substation power, which is calculated in step 4. It is also based on the working characteristics of the

supercapacitor (charging and discharging characteristics). The optimal capacity value of the supercapacitor is determined to achieve the goal of minimizing the total cost of power consumption and investment cost for the supercapacitor energy storage system as illustrated by the objective function:  $Min(energy\_cost(q)+sc\_cost(q))$ .

At the end of step 5, the optimal capacity value of the supercapacitor at each substation as well as the total cost value of operating the substation when installing the supercapacitor energy storage system with the optimal value, are determined.

Step 6. Calculating the operating cost for the railway line.

This step calculates the total cost for operating all substations in the railway line in the case of installing the optimal configuration of the supercapacitor. This total cost is defined for the union k. Finish step 6 then go back to step 3.

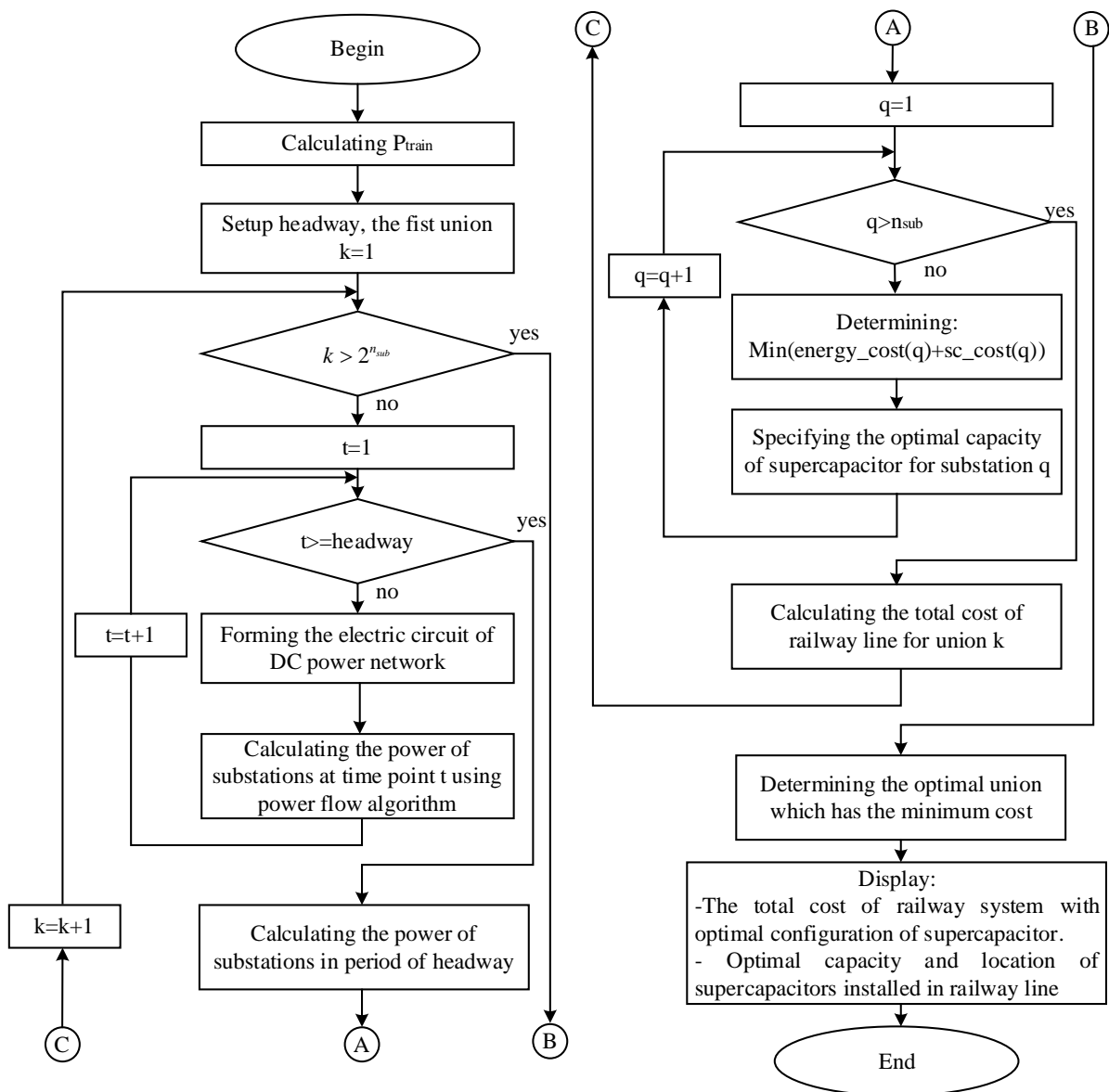


Figure 2. Flow chart of the algorithm for finding the optimal location and capacity of SESSs.

Step 7. Finding the optimal union of supercapacitor configuration with the lowest cost function value.

From the cost function value of each configuration that has been determined in this algorithm from step 4 to step 6, step 7 compares the cost function values of the unions and finds the union with the smallest value. This is the optimal union to look for.

The end of step 7 will show the results on the optimal location and capacity of the supercapacitor placed on the railway, as well as the optimal cost function value when installing the optimal configuration of the supercapacitor.

Step 8. End

Figure 2 presents the flow chart of the proposed algorithm.

## 4. SIMULATION RESULTS

### 4.1. Case study

To experiment with the proposed solution, simulations on a model of an urban railway line were carried out in MATLAB. The information of the simulating model is referred to in [13]. This railway has a total length of 12.6 km, on which there are 12 stations, and it is powered by 5 substations. Table 1 shows the main electrical and topological characteristics of this line, as well as the information regarding the rolling stock.

Table 1. The main electrical and topological characteristics.

Parameters	value
Line length	12.6 km
Station location	0; 0.93; 1.81; 2.89; 4.14; 5.15; 6.45; 7.75; 9.08; 10.18; 11.62; 12.60
Substation location	0; 2.89; 6.45; 9.08; 12.60
Nominal voltage	750 VDC
Train mass	1450 ton
Auxiliary power	200 kW
Maximum speed	80 km/h
Headway	300 seconds
Resistance per unit of contact line and track	0.065 $\Omega$ /km
Resistance of feeder	0.01 $\Omega$

The travel time of trains between stations and the whole route is described in Table 2. Assume that the departure time from the first station is 0 seconds, the rest time at the stations is assumed in 30 seconds. Then the total time for the journey from the first station to the last station takes 1292 seconds (about 21.5 minutes). The journey in the opposite direction took 1285 seconds.

The cost of the parameters for the calculation is listed in Table 3. The price of setting a supercapacitor energy storage system ( $Cost_{instal}$ ) is referenced in [14] (with a value of 5000 euros exchanged to VND will be 140425700 VND). Also, in this paper, the price of the



DC/DC converter ( $cost_{conv}$ ) and the maintenance price of the energy storage system ( $cost_{main}$ ) is 40 euros and 0.14 euros, respectively. The price of supercapacitor ( $cost_{sc}$ ) is referenced in [15] with an average value of 200 USD/kW.

Table 2. The journey time of the train along the railway line.

Departure station	Arrival station	Departure time (second)	Arrival time (second)	Journey time (second)	Rest time (second)
1	2	0	88	88	30
2	3	118	196	78	30
3	4	226	317	91	30
4	5	347	450	103	30
5	6	480	559	79	30
6	7	589	693	104	30
7	8	723	809	86	30
8	9	839	936	97	30
9	10	966	1050	84	30
10	11	1080	1181	101	30
11	12	1211	1292	81	30
12	11	1292	1372	80	30
11	10	1402	1503	101	30
10	9	1533	1617	84	30
9	8	1647	1744	97	30
8	7	1774	1859	85	30
7	6	1889	1995	106	30
6	5	2025	2103	78	30
5	4	2133	2237	104	30
4	3	2267	2355	88	30
3	2	2385	2464	79	30
2	1	2494	2577	83	30

Table 3. List of unit cost value.

Parameters	Value
Price of setting a supercapacitor energy storage system ( $cost_{instal}$ )	140425700 VND
Price of electricity ( $cost_{elec}$ )	3000 VND/kWh
Price of DC/DC converter ( $cost_{conv}$ )	1123400 VND/kW
Price of supercapacitor ( $cost_{sc}$ )	4611000 VND/kW
Price of DC/DC converter ( $cost_{conv}$ )	3932 VND/kW/day
Operating time in a day	16 hours
Annual operating days	365 days
The operating period of the railway line (ny)	10 years

#### 4.2. Numerical results

Based on the main electrical and topological characteristics described in Table 1 as well as the journey time of the train moving along the railway line as shown in Table 2, the train power at each time is estimated. To calculate the power according to formula (1), the traction force value and speed value are calculated according to the empirical method as mentioned in the reference [16]. Figure 3 illustrates the power-time graph of the train traveling along the route. Power is positive and increases with acceleration. This value decreases slightly when the train operates at a constant speed. In braking phases, the power will be negative. And the time when trains stop at the stations, the power will be zero.

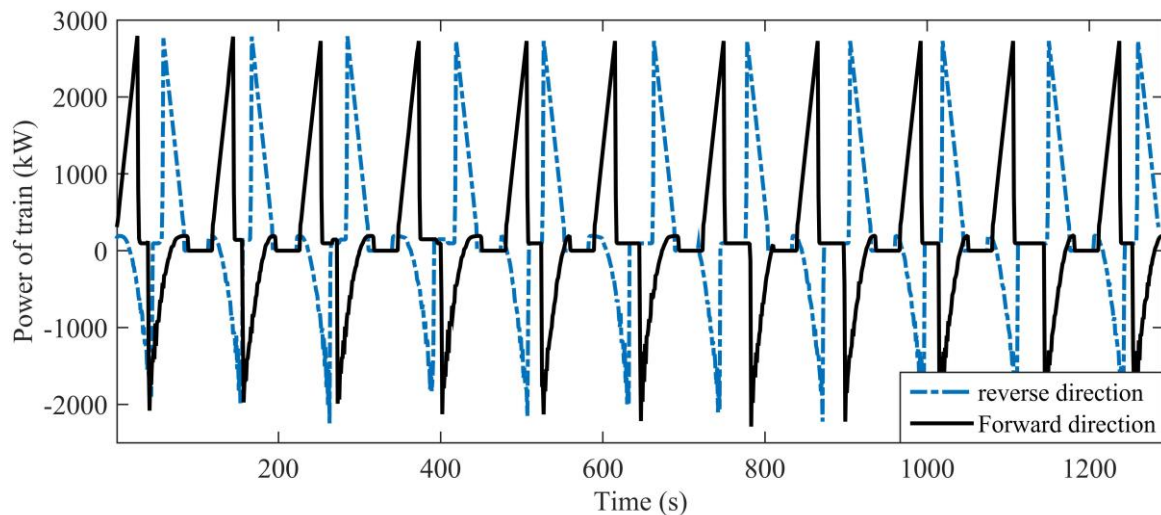


Figure 3. Power of train at each time point along the railway line.

In the case study, the headway value is selected by 300 seconds to simulate the operating mode on the railway line. At that time on the railway line simultaneously operated 9 trains. The timetable is illustrated in Figure 4.

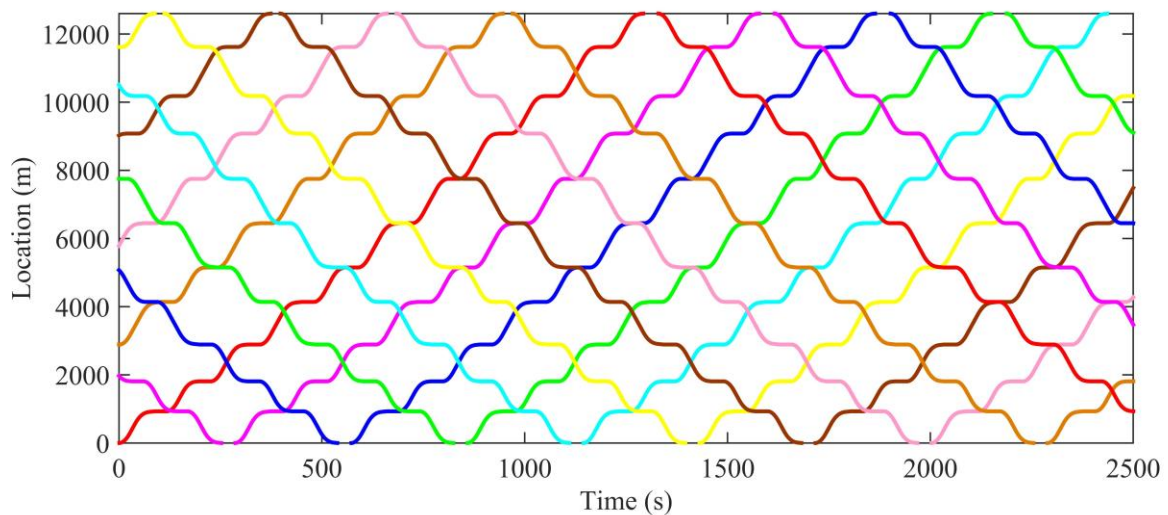


Figure 4. The timetable of operating scenario with headway = 300 (s).

Apply the power flow algorithm as developed in the reference [17] to determine the power value of substations at times in the headway duration, thereby determining the maximum energy level that can be reused. This recycled energy capacity is the basis value for determining the maximum capacity of supercapacitors that can be installed at substations. Figure 5 illustrates the power-time graph of the 5th substation. It can be seen that there are three time periods when the power of the substation is negative. During these stages, the supercapacitor performs charging mode to store energy. At the stages where the power is positive, the energy stored in the supercapacitor is used, and the supercapacitor works in discharge mode. Assume that the controlling mode for the supercapacitor according to the characteristic curve described in [18]. From there, the power value of the supercapacitor at each working time is determined.

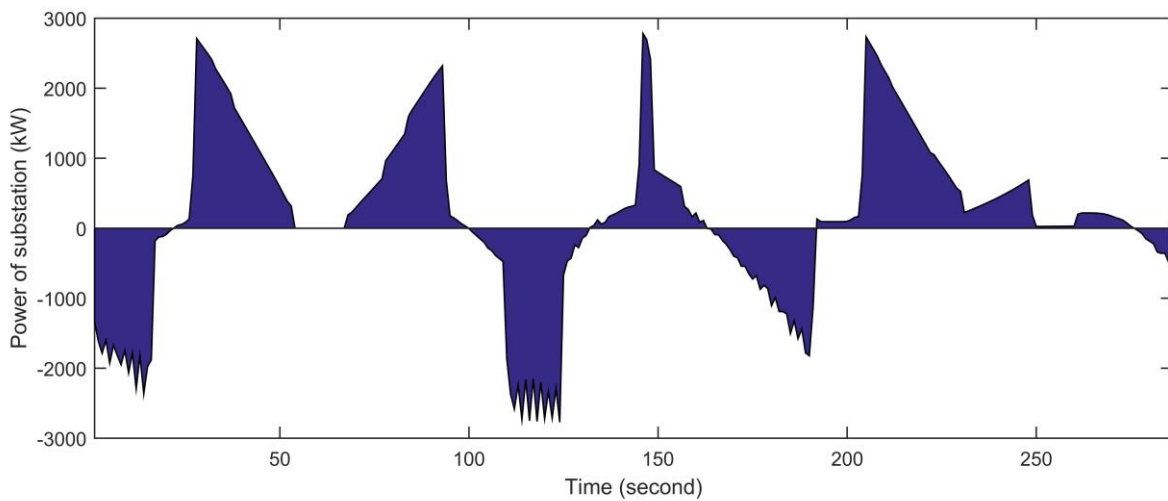


Figure 5. Power of the fifth substation at each time point.

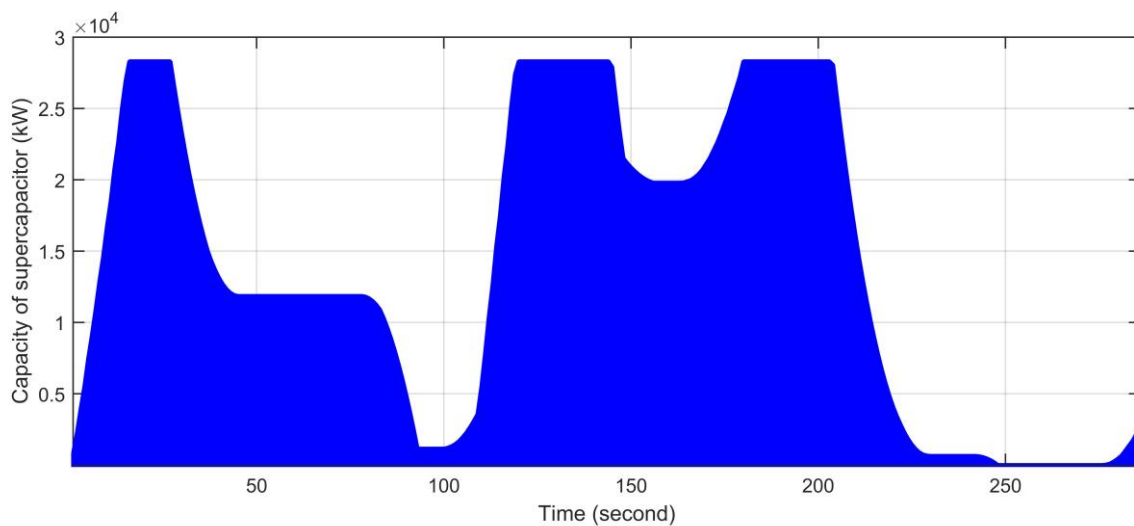


Figure 6. Power of supercapacitor installed in the fifth substation at each time point.

Figure 6 illustrates the power-time graph of the supercapacitor when installed at the 5th substation. From time 0 to 25 seconds the supercapacitor stores energy. In the period from 25 seconds to 100 seconds, the supercapacitor discharges to support the substation. The energy is evenly distributed according to the mechanism to reduce the load peaks for the substation, and also fully utilized for each discharge and charge cycle. The process is similar for subsequent charge and discharge cycles.

With the installation of supercapacitors at the substation, the power consumption of the substation will be improved. Figure 7 depicts the graph of power consumption of substations in two cases with and without installing the supercapacitor.

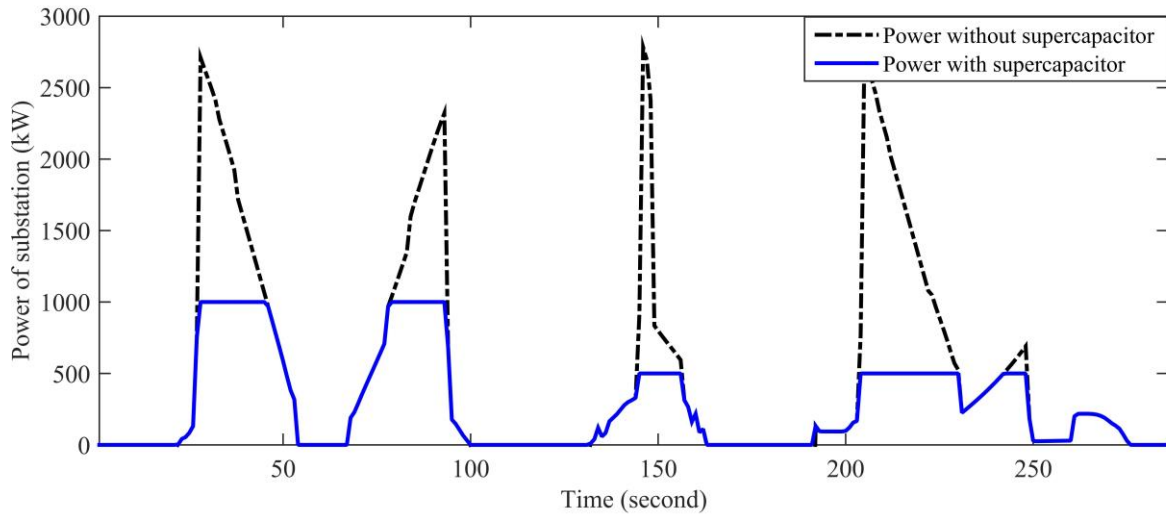


Figure 7. Consumption power of substation with and without installing supercapacitor.

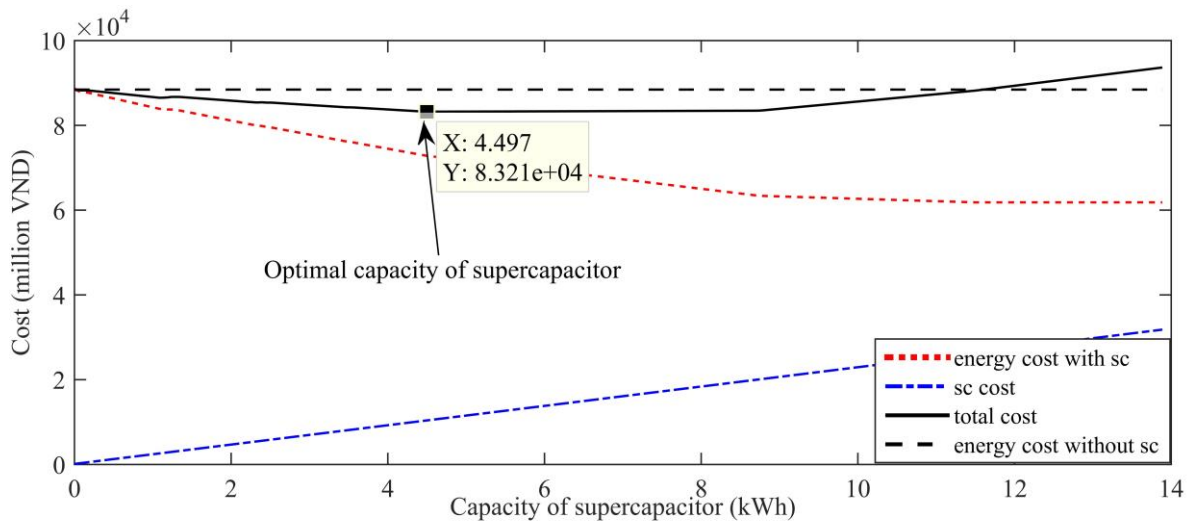


Figure 8. Relationship of cost functions with supercapacitor capacity.

After determining the total energy that can be recovered in each cycle, the algorithm executes to find the optimal capacity of the supercapacitor to ensure the objective function in formula (5). In which, the energy consumption cost of the traction power station is indirectly inversely proportional to the capacity of the supercapacitor. However, when the

Table 4. Total costs correspond to different supercapacitor installation configurations.

Union	Supercapacitor capacity (kWh)					Total cost (billion VND)
	Sub1	Sub2	Sub3	Sub4	Sub5	
1	1.21	1.62	1.76	1.70	6.95	440.29
2	1.21	1.62	1.76	6.59	0.00	410.72
3	1.21	1.62	1.77	0.00	7.15	422.26
4	1.21	1.62	3.14	0.00	0.00	387.06
5	1.21	0.75	0.00	2.33	6.97	432.43
6	1.21	0.75	0.00	3.89	0.00	405.43
7	1.21	1.29	0.00	0.00	9.23	411.67
8	1.21	4.95	0.00	0.00	0.00	369.27
9	2.45	0.00	2.28	1.70	6.95	412.47
10	2.45	0.00	2.29	6.59	0.00	383
11	2.45	0.00	1.90	0.00	7.15	394.72
12	2.45	0.00	4.23	0.00	0.00	360.59
13	2.58	0.00	0.00	2.39	6.97	402.1
14	2.58	0.00	0.00	4.09	0.00	375.39
15	2.68	0.00	0.00	0.00	9.14	382.09
<b>16</b>	<b>3.24</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>336.75</b>
17	0.00	3.51	1.76	1.70	6.95	431.26
18	0.00	3.51	1.76	6.59	0.00	401.69
19	0.00	3.51	1.77	0.00	7.15	413.23
20	0.00	3.51	3.14	0.00	0.00	378.06
21	0.00	3.51	0.00	2.33	6.97	424.32
22	0.00	3.51	0.00	3.89	0.00	397.33
23	0.00	4.34	0.00	0.00	9.23	404.09
24	0.00	5.32	0.00	0.00	0.00	360.1
25	0.00	0.00	2.75	1.70	6.95	395.57
26	0.00	0.00	2.75	6.59	0.00	366.32
27	0.00	0.00	2.80	0.00	7.15	380.48
28	0.00	0.00	5.46	0.00	0.00	349.63
29	0.00	0.00	0.00	2.80	6.97	382.88
30	0.00	0.00	0.00	5.37	0.00	358.09
31	0.00	0.00	0.00	0.00	9.01	360.56
32	0.00	0.00	0.00	0.00	0.00	460.099

supercapacitor capacity increases to a certain value corresponding to the maximum energy recoverable, the energy cost does not decrease anymore and will remain at a constant value. The investment, operation, and maintenance costs for a supercapacitor energy storage system are linearly proportional to the capacity of the supercapacitor. Thus, the total cost function has

a relationship with the capacity of the supercapacitor. With the help of a computer tool, the total cost function will be calculated corresponding to each value of the supercapacitor capacity. From there, the optimal capacity value of the supercapacitor is determined corresponding to the minimum value of the total cost function. Figure 8 depicts the relationship of the cost functions to the capacity of the supercapacitor. In the simulation case for the 5th substation, the optimal capacity of the supercapacitor is found to be 4,497 kWh, corresponding to the total cost function of 83.21 billion VND/10 years.

Table 5 summarizes the total cost function as well as the optimal capacity of supercapacitors corresponding to supercapacitor installation configurations at substations throughout the railway line. In the simulation model, there are 5 substations, so there are up to  $2^5 = 32$  supercapacitor installation configurations. Comparing the value of the total cost function, it can be seen that configuration number 16 reaches the smallest value corresponding to the supercapacitor installed only at substation No. 1 with a capacity of 3.24 kWh. The total cost of traction power within 10 years with the supercapacitor installation configuration for this case is 336.75 billion VND.

## 5. CONCLUSION

This paper presents a method to determine the optimal location and capacity of SESSs for urban railway lines with the aim of maximizing economic benefits. To solve the optimization problem with two dependent parameters, the proposed method fixes the location parameter then determining the remaining parameter. Because the location variable has a small space, the proposed method ensures feasibility, and always finds the global optimal solution. The proposed method can be used for any type of urban electrified railway line, it can model any existing rolling stock and allows to simulate any working timetables and the simultaneous presence of SESSs configuration. In this way, it is possible to find the optimal location and capacity of SESSs. The foundation of this study can provide a valuable reference to subway companies.

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