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MICROMECHANICAL APPROACH TO DETERMINE THE EFFECTS OF SURFACE AND INTERFACIAL ROUGHNESS IN MATERIALS AND STRUCTURE UNDER COSINUSOIDAL NORMAL PRESSURE

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Abstract. Contact mechanics is a topic that performs the investigation of the deformation of solids that touch each other at one or more points. A principal distinction in contact mechanics is between stresses acting perpendicular to the contacting bodies' surfaces and stresses acting tangentially between the surfaces. This study focuses mainly on the normal stresses that are caused by applied forces. As a case study, the present work aims at investigating the bi-dimensional contact mechanics of wavy cosinusoidal anisotropic finite planes. To achieve this objective, results on the displacement and stress component are first calculated with the help of the Lekhnitskii formalism. Then, with the application of normal pressure at plane surface and by applying boundary conditions at depth *h* of solid we obtain solution for the contact pressure in closed form. In case of infinite anisotropic plane where the depth h tends to infinite, by using results obtained with finite *h* we derive the analytical solution for vertical displacement at the surface. As an illustration, behaviour of a monoclinic material under consinusoidal pressure is analyzed.

Keywords: contact mechanics, anisotropic materials, Lekhnitskii formalism.

1. INTRODUCTION

In physics and mechanics of composite materials, most investigations dedicated to determining the behavior of contact mechanic often adopt the hypothesis that the surfaces are smooth. However, in many practical situations, the assumption of smooth surfaces is too idealized and the consideration of rough surfaces is unavoidable. Consequently, the real contact problem of two surfaces can be described by several stages: the surfaces approach and firstly touch each other at the peaks of their asperities, the asperities are then flattened and the contact areas spread as the load increases and finally the full contact status is reached at sufficiently large load. In order to understand the contacts at microscale throughout different stages, the roughness model plays a very important part.

Contact mechanics is the study of the deformation of solids that touch each other at one or more points [1-3]. Principles of contacts mechanics are implemented towards applications such as locomotive wheel-rail contact, coupling devices, braking systems, tires, bearings, combustion engines, mechanical linkages, gasket seals, metalworking, metal forming, ultrasonic welding, electrical contacts, and many others. And its application can extend in micro and nanotechnology [2, 8]. In fact, the problem of con- tact between the corrugated surface plays an important role. However, most of the previously mentioned works is limited to isotropic materials [9-11] wherea a large number of materials in nature exhibiting properties that vary with direction, this is the case of anisotropy. In this work, we aim investigate elastic problem with a cosinusoidal pression placed at surface of a finite solid made of a homogeneous anisotropic elastic material using the method of complex variables [4-7]. This paper is organized as follows: Section 2 describes the method of complex variable based on the Lekhnitskii formalism. In Section 3 and 4, we show how to obtain displacement and stress field from a given periodical traction at surface in case of finite and infinite anisotropic plane from a given periodical traction at surface. Numerical examples of analytical results obtained by method of complex variable are illustrated in section 5. Finally, a few concluding remarks are shown in Section 6

2. THE LEKHNITSKII FORMALISM

We consider a solid which consists of a linearly elastic anisotropic homogeneous material and under- goes plane strains in the plane xOy. The material is considered monoclinic with symmetry plane as deformation plane. The corresponding stress-strain relation of the material is given by the Hooke law

$$\begin{cases} \sigma_{xx} = L_{11}\varepsilon_{xx} + L_{12}\varepsilon_{yy} + 2L_{16}\varepsilon_{xy}, \\ \sigma_{yy} = L_{12}\varepsilon_{xx} + L_{22}\varepsilon_{yy} + 2L_{26}\varepsilon_{xy}, \\ \sigma_{xy} = L_{16}\varepsilon_{xx} + L_{26}\varepsilon_{yy} + 2L_{66}\varepsilon_{xy}, \\ \sigma_{zz} = L_{13}\varepsilon_{xx} + L_{23}\varepsilon_{yy} + 2L_{66}\varepsilon_{xy}, \\ \qquad \sigma_{yz} = 0, \;\; \sigma_{xz} = 0. \end{cases} \tag{1}$$

where$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$and $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}$ are the stress and strain components, L_{ij} (i, j = 1, 2, 3, 6) presents the reduced elastic stiffness associated to a plane strain problem [6]. By resolving Eq.(1) we can deduce the stress-strain relation as follows:

$$\begin{cases} \varepsilon_{xx} = S_{11}\sigma_{xx} + S_{12}\sigma_{yy} + 2S_{16}\sigma_{xy}, \\ \varepsilon_{yy} = S_{12}\sigma_{xx} + S_{22}\sigma_{yy} + 2S_{26}\sigma_{xy}, \\ 2\varepsilon_{xy} = S_{16}\sigma_{xx} + S_{26}\sigma_{yy} + 2S_{66}\sigma_{xy}, \end{cases} \quad (2)$$

where S_{ij} stand for the reduced elastic compliances associated to a plane strain problem [12] are in function of L_{ij} . In the absence of body forces, for plane strain, the equilibrium equations is written as:

$$\begin{cases} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0, \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0. \end{cases} \quad (3)$$

It is observed that these equations will be identically satisfied by choosing a representation

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}. \quad (4)$$

where $\phi = \phi(x, y)$ is an arbitrary form called the Airy stress function [4, 6]. With regard to strain compatibility for plane strain, the Saint-Venant relations reduce to

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}. \quad (5)$$

By substituting Eqs. (2, 4) into Eq. (5) we obtain:

$$S_{22}\frac{\partial^4 \phi}{\partial x^4} - 2S_{66}\frac{\partial^4 \phi}{\partial x^3 \partial y} + (2S_{12} + S_{66})\frac{\partial^4 \phi}{\partial x^2 \partial y^2} - 2S_{16}\frac{\partial^4 \phi}{\partial x \partial y^3} + S_{11}\frac{\partial^4 \phi}{\partial y^4} = 0 \quad (6)$$

According to the formalism of Lekhnitskii [4 ,6], the stress and displacement fields in the anisotropic solid are determined by two complex potential functions $\phi_1(z_1)$ and $\phi_2(z_2)$ of complex variables z_1 and z_2:

$$z_1 = x + \mu_1 y, \ z_2 = x + \mu_2 y. \quad (7)$$

In these expressions, the constants μ_1 and μ_2 are two complex roots of the characteristic equation

$$S_{11}\mu^4 - 2S_{16}\mu^3 + (2S_{12} + S_{66})\mu^2 - 2S_{26}\mu + S_{22} = 0. \quad (8)$$

Since Eq.(8) is of order 4 with real coefficients, it has two pairs of conjugate roots. With no loss of generality, we choose μ_1 and μ_2 to be the two roots having positive imaginary (I) parts

$$\begin{cases} I(\mu_1) > 0, \\ I(\mu_2) > 0. \end{cases} \quad (9)$$

To within a rigid displacement, the displacement components, u along x and v along y, are provided by

$$\begin{cases} u(x, y) = 2R[p_1\phi_1(z_1) + p_2\phi_2(z_2)], \\ v(x, y) = 2R[q_1\phi_1(z_1) + q_2\phi_2(z_2)]. \end{cases} \quad (10)$$

where

$$\begin{cases} p_i = S_{11}\mu_i^2 - S_{16}\mu_i + S_{12}, \\ q_i = S_{12}\mu_i - S_{26} + \frac{S_{22}}{\mu_i}. \end{cases} \quad (11)$$

At the same time, the stress components $\sigma_{xx}, \sigma_{yy}\ and\ \sigma_{xy}$ are delivered by

$$\begin{cases} \sigma_{xx}(x,y) = 2R[\mu_1^2\phi_1'(z_1) + \mu_2^2\phi_2'(z_2)], \\ \sigma_{yy}(x,y) = 2R[\phi_1'(z_1) + \phi_2'(z_2)] \qquad , \\ \sigma_{xy}(x,y) = -2R[\mu_1\phi_1'(z_1) + \mu_2\phi_2'(z_2)]. \end{cases} \quad (12)$$

where ϕ_1' and ϕ_2' are derivatives of ϕ_1 and ϕ_2 respectively, and R stands for the real part of function.

3. PERIODICAL TRACTION ON A FINITES ANISOTROPIC PLANE

Consider an anisotropic solid where thickness is h. At the surface of a finite anisotropic plane a cosinusoidal normal pressure p(x) of wave length and amplitude p, namely

$$p(x) = p^* cos\left(\frac{2\pi x}{\lambda}\right), \quad (13)$$

is applied.

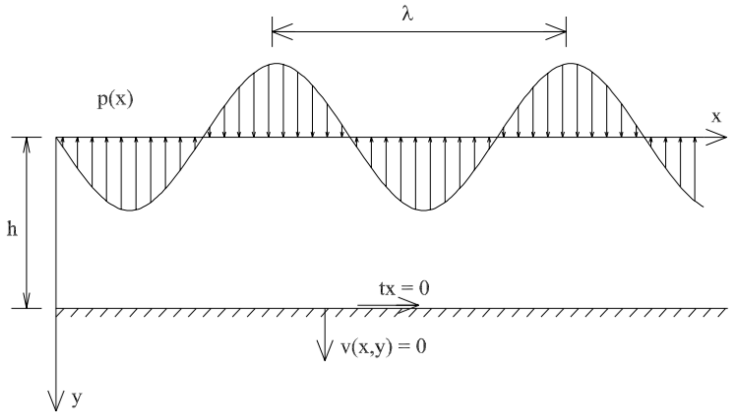


Figure 1. Cosinusoidal normal pressure applied at surface of solid and boundary conditions

At depth h, we block the vertical displacement v (x, h) = 0, and the solid can move horizontally without friction $\sigma_{xy}(x,y) = 0$. Accounting for the boundary condition Eq. (13), we propose the following complex potential functions

$$\begin{cases} \phi_1(z_1) = \frac{A_1\lambda p^*}{4\pi i} exp\left(\frac{2i\pi z_1}{\lambda}\right) + \frac{B_1\lambda p^*}{4\pi i} exp\left(\frac{-2i\pi z_1}{\lambda}\right) \\ \phi_2(z_2) = \frac{A_2\lambda p^*}{4\pi i} exp\left(\frac{2i\pi z_2}{\lambda}\right) + \frac{B_2\lambda p^*}{4\pi i} exp\left(\frac{-2i\pi z_2}{\lambda}\right)' \end{cases} \quad (14)$$

where A_j, B_j, μ_j, q_j (j = 1, 2) are complex numbers such as

$$A_j = a_j + i\alpha_j, B_j = b_j + i\beta_j, \mu_j = m_j + in_j, q_j = k_j + il_j \quad (15)$$

with $a_j, b_j, \alpha_j, \beta_j, m_j, n_j, k_j, l_j$ denotes real numbers, i is equal to $\sqrt{-1}$.

By substituting Eq. (15) into Eq. (10), solution of displacement field of half plan are expressed by:

$$u(x,y)=R\begin{bmatrix} \frac{p_1A_1\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_1}{\lambda}\right)+\frac{p_1B_1\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_1}{\lambda}\right) \\ +\frac{p_2A_2\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_2}{\lambda}\right)+\frac{p_2B_2\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_2}{\lambda}\right) \end{bmatrix}, \quad v(x,y)=R\begin{bmatrix} \frac{q_1A_1\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_1}{\lambda}\right)+\frac{q_1B_1\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_1}{\lambda}\right) \\ +\frac{q_2A_2\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_2}{\lambda}\right)+\frac{q_2B_2\lambda p^*}{2\pi i}exp\left(\frac{2i\pi z_2}{\lambda}\right) \end{bmatrix} \tag{16}$$

and by substituting Eq. (15)into Eq. (12) solution for stress fields are defined by

$$\begin{aligned} \sigma_{xx}(x,y) &= p^*\Re\left[\mu_1^2A_1exp\left(\frac{2i\pi z_1}{\lambda}\right)-\mu_1^2B_1exp\left(\frac{-2i\pi z_1}{\lambda}\right)+\mu_2^2A_2exp\left(\frac{2i\pi z_2}{\lambda}\right)-\mu_2^2B_2exp\left(\frac{2i\pi z_2}{\lambda}\right)\right] \\ \sigma_{yy}(x,y) &= p^*\Re\left[A_1exp\left(\frac{2i\pi z_1}{\lambda}\right)-B_1exp\left(\frac{-2i\pi z_1}{\lambda}\right)+A_2exp\left(\frac{2i\pi z_2}{\lambda}\right)-B_2exp\left(\frac{2i\pi z_2}{\lambda}\right)\right] . \\ \sigma_{xy}(x,y) &= -p^*\Re\left[\mu_1A_1exp\left(\frac{2i\pi z_1}{\lambda}\right)-\mu_1B_1exp\left(\frac{-2i\pi z_1}{\lambda}\right)+\mu_2A_2exp\left(\frac{2i\pi z_2}{\lambda}\right)-\mu_2B_2exp\left(\frac{2i\pi z_2}{\lambda}\right)\right] \end{aligned} \tag{17}$$

Displacement and stress fields solution of solid are defined by determining four unknowns A_1, A_2, B_1, B_2. In the following paragraphs we consider two boundary conduction problems applied to solid:

- At the plane surface y = 0 ($z_1 = z_2 = x$):

$$T=\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{yx} & \sigma_{yy}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\ p^*cos\left(\frac{2\pi x}{\lambda}\right)\end{bmatrix} \tag{18}$$

- At depth y = h:

$$\begin{cases} v(x,h)=0, \\ t_x=\sigma_{xy}(x,h)=0. \end{cases} \tag{19}$$

by substituting Eqs. (16, 17) in boundary equations Eqs. (18, 19) and requiring the real and imaginary part of equations to be equal to zero we obtain a system of eight equations with eight unknows a_1, a_2, b_1, b_2, α_1, α_2, β_1, β_2. By solving this system of equations, we deduce eight unknows which are components of unknows complex A_1, A_2, B_1, B_2.

By replacing solution of $\phi_1(z_1)$ and $\phi_2(z_2)$ in Eq. (16_2) derive the expression of vertical displacement at the surface:

$$v(x,0)=2\Re\left[q_1\phi_1(x)+q_2\phi_2(x)\right]=\frac{\lambda p^*}{2\pi}\left[Hcos\left(\frac{2\pi x}{\lambda}\right)+Ksin\left(\frac{2\pi x}{\lambda}\right)\right] . \tag{20}$$

where H =I (A1 +A2 +B1 + B2) and K = $\Re$ (A1 + A2 − B1 − B2). It is interesting to remark that a harmonic surface traction generates a harmonic surface displacement of the same wavelength. But a phase shift occurs due to the sinus term in the right-hand side of Eq. (20). This result, which seems been reported in the literature [8] for case of anisotropic half plane, and is in contrast to what happens in the case where the material forming the half plane is isotropic [1]. The phase shift disappears if and only if K = 0.

4. PERIODICAL TRACTION ON A INFINITE ANISOTROPIC PLANE

In case where y = h tends to ∞, the complex potential functions presented by Eqs. (14) are reduced to:

$$\phi_1(z_1) = \frac{A_1\lambda p^*}{4\pi i}\exp\left(\frac{2i\pi z_1}{\lambda}\right),$$
$$\phi_2(z_2) = \frac{A_2\lambda p^*}{4\pi i}\exp\left(\frac{2i\pi z_2}{\lambda}\right). \quad (21)$$

When a normal consinusoidal pressure proposed by Eq. (13) is applied at surface (y = 0), by substituting Eq. (21) into Eq. (12) we have:

$$\sigma_{yy}(x,0) = p^*\left[\Re(A_1 + A_2)\cos\left(\frac{2\pi x}{\lambda}\right) - I(A_1 + A_2)\sin\left(\frac{2\pi x}{\lambda}\right)\right],$$
$$\sigma_{xy}(x,0) = -p^*\left[\Re(A_1\mu_1 + A_2\mu_2)\cos\left(\frac{2\pi x}{\lambda}\right) - I(A_1\mu_1 + A_2\mu_2)\sin\left(\frac{2\pi x}{\lambda}\right)\right]. \quad (22)$$

Boundary conditions at surface requiring that:

$$\begin{cases} \sigma_{xy}(x,0) = 0, \\ \sigma_{yy}(x,0) = p^*\cos\left(\frac{2\pi x}{\lambda}\right). \end{cases} \quad (23)$$

By solving the system of equations Eqs. (23) yields:

$$A_1 = \frac{\mu_2}{\mu_2-\mu_1}, \quad A_2 = \frac{\mu_2}{\mu_2-\mu_1}. \quad (24)$$

Now we are interesting to determine the vertical displacement at surface. Introducing Eqs. (21) together with (24) into Eq. (16) gives the vertical displacement at the surface

$$v(x) = v(x,0) = \frac{\lambda p^*}{2\pi}\left[H_1\cos\left(\frac{2\pi x}{\lambda}\right) + K_1\sin\left(\frac{2\pi x}{\lambda}\right)\right], \quad (25)$$

with

$$H_1 = I\left[\frac{q_1\mu_2}{\mu_2-\mu_1} + \frac{q_2\mu_1}{\mu_1-\mu_2}\right], K_1 = \Re\left[\frac{q_1\mu_2}{\mu_2-\mu_1} + \frac{q_2\mu_1}{\mu_1-\mu_2}\right]. \quad (26)$$

by inserting Eq. (11) into Eq. (26_2), it derive the explicit formula of K_1:

$$K_1 = \Re\left\{\frac{\mu_1\mu_2}{\mu_2-\mu_1}\left[S_{22}\left(\frac{1}{\mu_1^2} - \frac{1}{\mu_2^2}\right) - S_{26}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)\right]\right\} = S_{22}\Re\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right] - S_{26}. \quad (27)$$

On the other hand, the polynomial equation Eq. (8) have four complex solution $\mu_1, \mu_2, \mu_3, \mu_4$ and according to Sadd [6] between them there are relations:

$$\begin{cases} \mu_1\mu_2\mu_3\mu_4 = \frac{S_{22}}{S_{11}}, \\ \mu_1\mu_2\mu_3 + \mu_2\mu_3\mu_4 + \mu_1\mu_3\mu_4 + \mu_1\mu_2\mu_4 = 2\frac{S_{26}}{S_{11}}, \\ \mu_1\mu_2 + \mu_2\mu_3 + \mu_3\mu_4 + \mu_4\mu_1 + \mu_1\mu_3 + \mu_2\mu_4 = \frac{2S_{12}+2S_{26}}{S_{11}}, \\ \mu_1 + \mu_2 + \mu_3 + \mu_4 = 2\frac{S_{16}}{S_{11}}. \end{cases} \quad (28)$$

By dividing Eq (28_1) by Eq (28_2) we get

$$\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3} + \frac{1}{\mu_4} = 2\frac{S_{26}}{S_{22}}, \quad (29)$$

which is equivalent to

$$\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\overline{\mu_1}} + \frac{1}{\overline{\mu_2}} = 2\frac{S_{26}}{S_{22}}, \quad (30)$$

By taking the real value of two sides of Eq. (30) we obtain:

$$\Re\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) = \frac{S_{26}}{S_{22}} \quad (31)$$

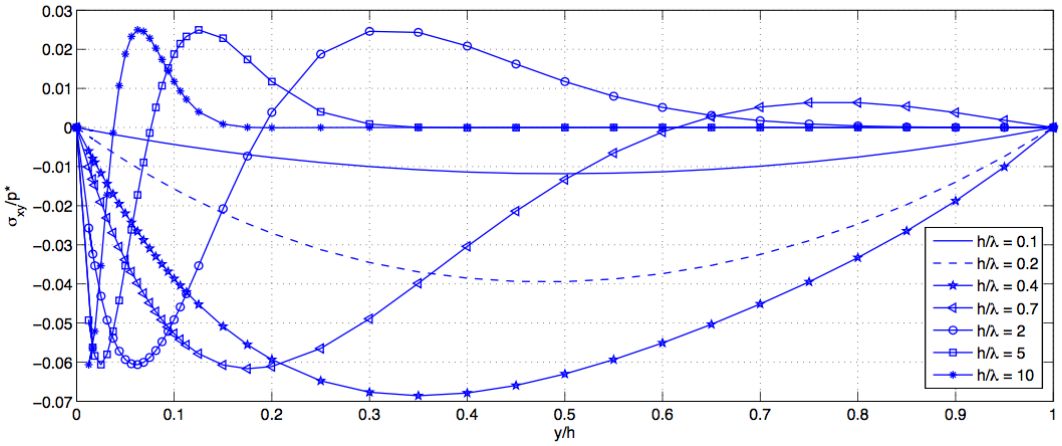
Replacing Eq. (31) into Eq. (27) we find that $K_1 = 0$, therefore

$$v(x,0) = \frac{\lambda p^*}{2\pi} \mathrm{H} \cos\left(\frac{2\pi x}{\lambda}\right). \quad (32)$$

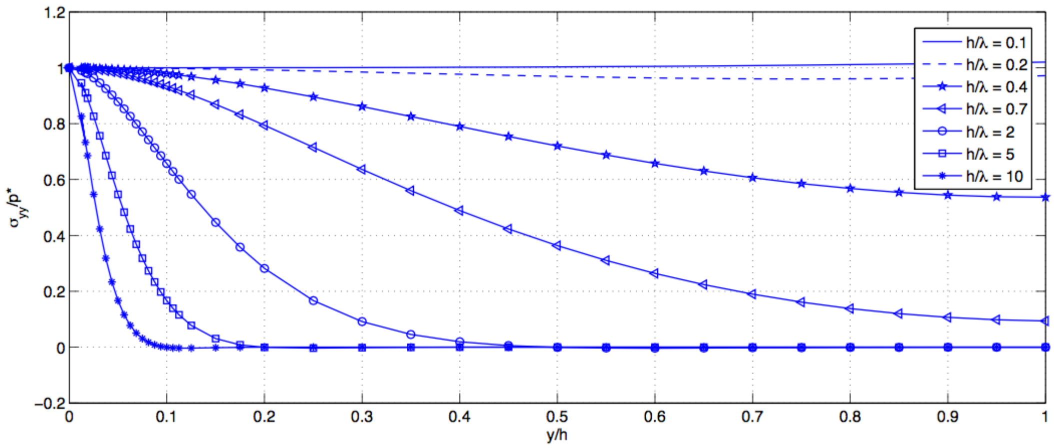
By comparing Eqs. (32) (13) it is interesting to emphasized that , if we apply a cosinusoidal surface traction at a surface of an infinite solid, it generates a periodic vertical displacement of the same wavelength and same phase as the pressure applied regardless of the anisotropy of the solid.

5. NUMERICAL EXAMPLES

To illustrate the analytical results presented above, we consider a monoclinic material $NaAlSiO_3$ whose the elastic constants in their plane of symmetry are given [13] $\mathbf{L} = \begin{bmatrix} 18.6 & 7.1 & 1.0 \\ 7.1 & 23.4 & 2.1 \\ 1.0 & 2.1 & 5.1 \end{bmatrix} 10^{11} MPa$. Variations in the values of normalized stress components σ_{xy} (x = 0, y) , σ_{yy} (x = 0, y) with respect to the amplitude p*, and variation in the value of normalized displacement u (x = 0, y), v (x = 0, y) with respect to wavelength λ, versus the value of fraction $\frac{y}{h}$ are plotted for different values of the ratio $\frac{h}{\lambda}$ in Fig 2 and Fig 3 respectively.



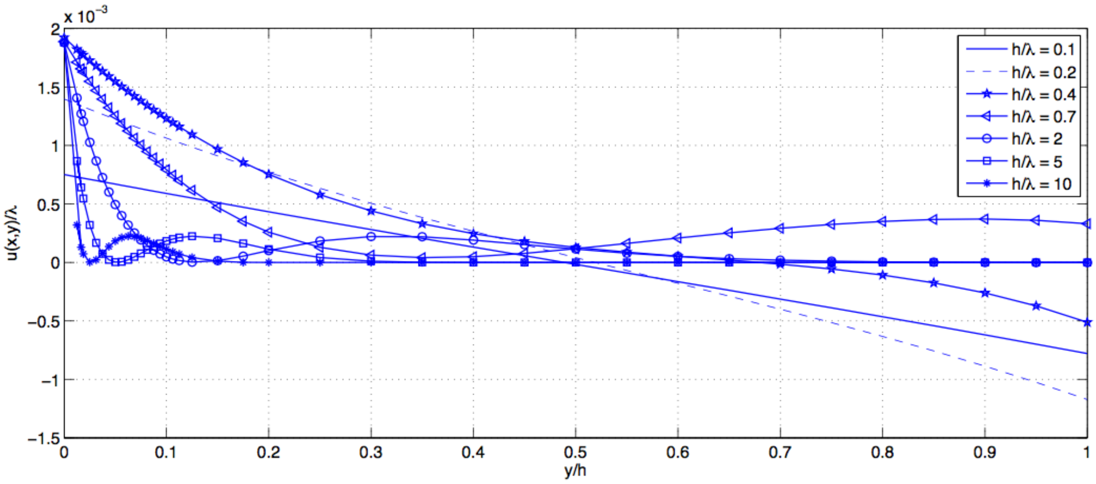
(a) Variation of $\frac{\sigma_{xy}}{p^*}$ in function of $\frac{y}{h}$



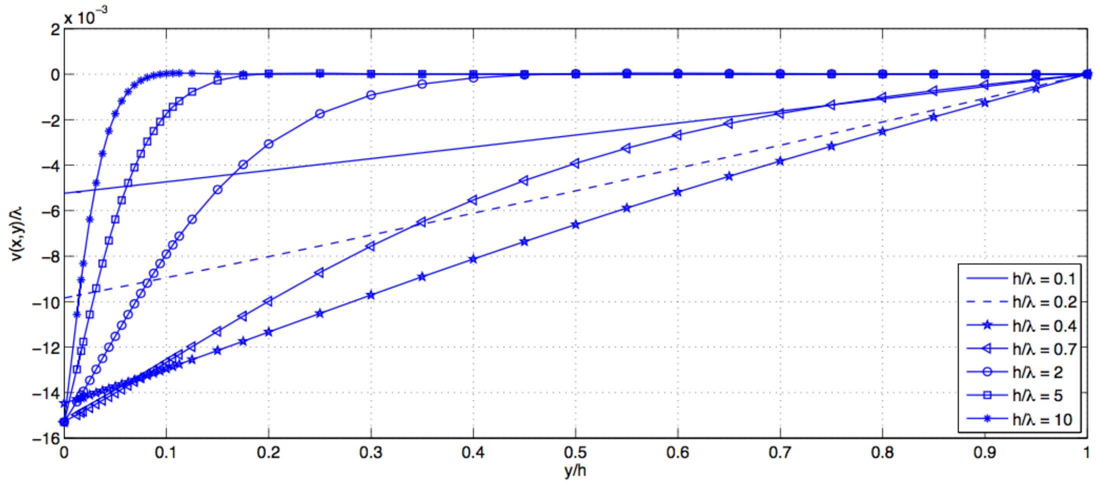
(b) Variation of $\frac{\sigma_{yy}}{p^*}$ in function of $\frac{y}{h}$

Figure 2. Variation of $\frac{\sigma_{\alpha\beta}}{p^*}$ versus $\frac{y}{h}$ with different ratios $\frac{h}{\lambda}$.

By considering the Fig 2.a, we contaste that with value of fraction $\frac{h}{\lambda}$ less than 0.4 the absolute maximum values of σ_{xy} are in negative zone, while if $\frac{h}{\lambda}$ is more than 0.7 then σ_{xy} has two maximum values at positive and negative zone. With bigger value of $\frac{h}{\lambda}$, σ_{xy} tends to zero at smaller value of $\frac{y}{h}$. Fig 2b demonstrate that value of σ_{yy} decrease with increase of depth y, and with bigger value of $\frac{h}{\lambda}$, σ_{yy} tends to zero at smaller value of $\frac{y}{h}$.



(a) Variation of $\frac{u_{xy}}{\lambda}$ in function of $\frac{y}{h}$



(b) Variation of $\frac{v_{xy}}{\lambda}$ in function of $\frac{y}{h}$

Figure 3. Variation of normalized displacement with respect to λ versus $\frac{y}{h}$ with different ratios $\frac{h}{\lambda}$.

Fig 3a proves that the increased value of $\frac{h}{\lambda}$ lead to decrease in u_{xy} at smaller value $\frac{y}{h}$. And u_{xy} tends to zero at depth y = 0.5h. As the value $\frac{h}{\lambda}$ increases, v_{xy} tend to zero at smaller value y are illustrated in Fig 3b. Difference from other values, if $\frac{h}{\lambda} = 0.1\ or\ 0.2$, v_{xy} increases linearly with the value of $\frac{y}{h}$.

6. CONCLUSION

The present work analyzes the two-dimensional elastic problems of anisotropic half planes supported a cosinusoidal traction applied at surface. The micromechanical approach used in the paper is method based on complex variable proposed by Sadd by applying the Lekhnitskii formalism. By using this method, we can determine the local solutions of displacement and then strain or stress fields of solid. Analytical results obtained in this paper show that: regardless of the anisotropy of the solid if we apply a cosinusoidal traction at the surface, it yields a harmonic displacement at same period with a dephase at surface in case of finite solid, but and in special case of infinite solid it generates a harmonic displacement at same period and phase shift is vanishing at infinity.

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