



## COMPARISON OF INELASTIC MOMENT RESISTANCES OF ROLLED STEEL BEAMS BASED ON DIFFERENT SPECIFICATIONS AND A NUMERICAL STUDY

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**Abstract.** The inelastic buckling resistances of wide flange beams are strongly influenced by residual stresses and initial imperfections. However, the resistances as evaluated from simple solutions presented in several popular design specifications are found to be considerably different. The present study thus develop a numerical solution in ABAQUS software to investigate the inelastic buckling moment resistances of rolled steel beams with compact sections and subjected to the effects of residual stresses and initial imperfections. The residual stresses are taken as provided in AISC, CSA S16, EC 3 specifications, while the initial imperfections are taken as the first lateral-torsional buckling mode with a magnitude limited in AISC, CSA specifications. Through comparisons between the specifications and the numerical solutions, one observes a significant difference between the moment resistances predicted by the specifications, in which the AISC predicts the highest values, while the EC 3 predicts the lowest moments. The moment resistances based on the present numerical models lie between the EC 3 and CSA solutions and they are relatively close to EC 3 solutions. Effects of load height positions on the inelastic buckling moment resistances are significant, as investigated in the present study.

**Keywords:** inelastic buckling, moment resistance, load height position, residual stresses, initial imperfection, buckling moment comparison

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### 1. INTRODUCTION

In the design of steel members for flexure, factored moment resistances are determined based on different design equations depending on the unbraced length of the members (e.g.,

AISC [1], CSA S16 [2]). The moment resistances are controlled by a fully plasticized moment for short beams, while they are controlled by elastic lateral-torsional moment resistances for long unbraced spans. For steel beams with intermediate spans, their moment resistances are normally controlled by inelastic lateral-torsional moment resistances (AISC [1], CSA S16 [2], EC 3 [3]). As indicated in specifications [1-4] and many studies [4-21], the inelastic resistances are significantly influenced by residual stresses and initial imperfections (or initial out-of-straightness). Residual stress distributions on steel cross-sections depend on the manufacturing and processing conditions. They are usually classified into welded- and rolled-based models [6-14] while initial imperfections depend on the manufacturing conditions [1-4].

Residual stresses of a welded section often have a complicated distribution in which residual stresses are highly localized at the weld areas (e.g., [6-11]), while the residual stresses of a rolled section often have a typical form as provided in Fig. 2a (e.g., [1-7]). It is noted that specifications AISC [1] and CSA S16 [2] seem not to distinguish the difference of residual stresses of welded and rolled sections [5], while specification EC 3 [3] account for the difference. The effect of residual stresses on the inelastic lateral-torsional buckling resistance has been widely studied. Mupeta et al [5] conducted a study to evaluate the different effects of residual stresses and initial imperfections on inelastic buckling resistances according to EC 3 [3]. Kabir và Bhowmick [5] performed a numerical study on the inelastic moment resistances of a slender beam under a uniform moment and different welded and rolled residual stress models. The study showed that the effect of residual stresses on buckling resistances was significant and it decreased the system capacity. Also, the resistances in [5] were successfully validated against experiment results, but they are lower than those provided by specification CSA S16 [2]. The study thus indicated that specifications AISC [1] and CSA S16 [2] overestimated the inelastic moment resistances. Dibley [8] presented an experimental study and a regression-based equation to determine inelastic lateral-torsional moment resistances of DL30 steel members subjected to uniform bending and a British Standard 15-based residual stress model (with a uniform distribution of residual stresses on the section web). Other similar discussions of the effect of residual stresses on the buckling resistance of steel members were presented in studies [9-12].

For the effect of imperfection, both specifications AISC and CSA S16 [1,2] accept an allowable initial imperfection of  $L/1000$  in which  $L$  is the unbraced length of the member, while EC3 [3] established analytical solutions for the inelastic buckling moment resistances based on different levels of initial imperfection (as provided in Table 5.1 of Eurocode 3 specification [3,5]). Meanwhile, the current Vietnamese specification [4] seems not to cover the effect of initial imperfections. The effects of initial imperfection on the inelastic lateral torsional buckling of beam members under flexure were reported in many studies (e.g., [13-18]). Abebe et al. [13] presented an inelastic buckling analysis of steel columns, while Elaiwi et al [14] developed numerical solutions for castellated beams. Timoshenko and Gere [15], Trahair et al [16], Galambos [17] and Ziemian [12] had books about the design of steel members under the effect of initial imperfections. However, the documents did not consider the combination of both residual stresses and initial imperfections on inelastic buckling resistances. Couto and Real [18] conducted a numerical investigation on the influence of imperfections in the lateral-torsional buckling of beams with slender I-shaped welded sections.

Based on the present context, the present study will fill in the gap by conducting a comparison study on the inelastic moment resistances of wide flange steel beams with a

compact cross-section under the effect of both rolled-based residual stresses initial imperfections. Equations to evaluate the inelastic buckling resistances based on four typical specifications [1,2,3,4] are first summarized and discussed. Then, a numerical solution developed in ABAQUS [19] is going to be developed to determine the inelastic moment resistances. Because specifications often have different treatments to establish equations of buckling resistances, the present study is first going to compare the differences of the moment resistances between the four specifications [1,2,3,4] and the developed numerical solution. The effect of load positions on the cross-section may also change the inelastic moment resistances, thus they are going to be investigated in the present study. Through the comparisons, discussions of the characteristics of the specifications and the numerical study are finally clarified.

## 2. STATEMENT OF THE PROBLEM

A simply supported beam subjected to a midspan point load  $P$  is considered (Fig. 1a). The beam is laterally unsupported, it has a span of  $L$  ( $L=3,4$  or  $5$ m) and a prismatic  $W250 \times 45$  cross-section [2]. The section meets compact conditions according to AISC [1] and class 1 according to CSA S16 and EC 3 [2,3]. Three cases of load positions (i.e., top, middle and bottom positions) are considered (Figs. 1b,c,d). Residual stresses of rolled steels are included (Fig. 2a) and steel is assumed as a perfectly plastic material with an elastic modulus of  $E=200GPa$ , a yielding strength of  $F_y=350MPa$  and a Poisson's ratio of 0.3 (Fig. 2b). A numerical solution is developed in the present study and it captures both residual and imperfection effects, in which the magnitude of the imperfection is taken as  $L/1000$  [1,2,5]. It is required to evaluate and compare the inelastic lateral-torsional moment resistances based on specifications [1-4] and the numerical solution developed in the present study.

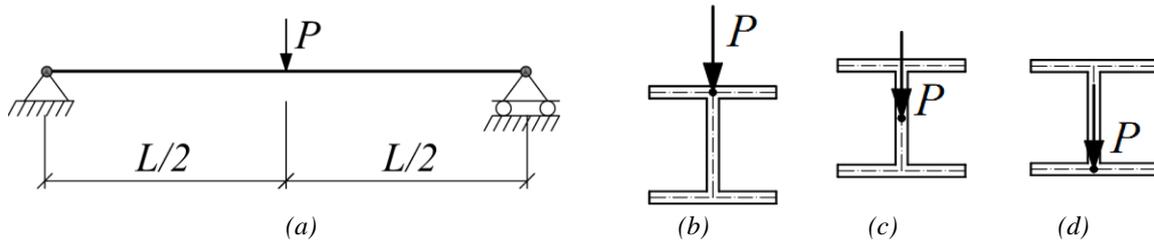


Figure 1. Description of the problem (a) beam profile, (b,c,d) Positions of load  $P$  on the cross-section

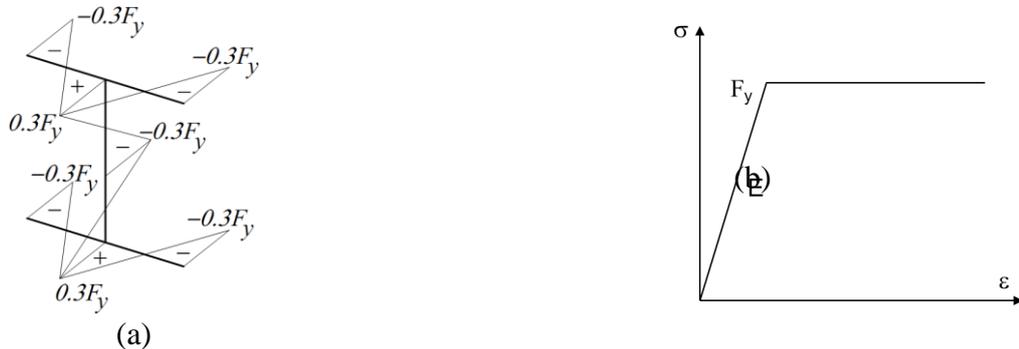


Figure 2. Assumptions of the rolled steel (a) residual stresses and (b) stress-strain relationship

### 3. MOMENT RESISTANCE BASED ON AISC A360-16 SPECIFICATION [1]

The AISC [1] specification provides formulas to determine the lengths  $L_p$  and  $L_r$  to distinguish the limits of the plasticized zone, inelastic buckling zone and elastic buckling zone. The limits can be evaluated as

$$L_p = 1.76r_y \sqrt{\frac{E}{F_y}} \quad (1)$$

$$L_r = 1.95r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{J}{S_x h_o} + \sqrt{\left(\frac{J}{S_x h_o}\right)^2 + 6.76 \left(\frac{0.7F_y}{E}\right)^2}} \quad (2)$$

$$r_{ts}^2 = \frac{\sqrt{I_y I_w}}{S_x} \quad (3)$$

where  $r_y$  is radius of gyration about  $y$ -axis,  $E$  is modulus of elasticity of steel,  $J$  is the St. Venant torsional constant,  $I_y$  is the flexural moment of inertia about the weak axis of the section,  $S_x$  is elastic section modulus taken about the  $x$ -axis, and  $h_o$  is the distance between the flange centroids,  $I_w$  is the warping torsional constant of the section.

For the present doubly symmetric compact I-shaped sections ( $W250 \times 45$ ) and the beam is laterally unsupported, the factored moment resistance  $M_r$  is based on the plasticized moment resistance when  $L_b \leq L_p$  as follows

$$M_r = \phi M_p = \phi Z F_y \quad (4)$$

where  $Z$  is the plastic section modulus. When  $L_p < L_b \leq L_r$ , resistance  $M_r$  is based on inelastic lateral-torsional buckling strength, i.e.,

$$M_r = \phi C_b \left[ M_p - (M_p - 0.7F_y S_x) \frac{L_b - L_p}{L_r - L_p} \right] \leq M_p \quad (5)$$

and when  $L_b > L_r$ , resistance  $M_r$  is based on elastic lateral-torsional buckling strength, i.e.,

$$M_r = \phi F_{cr} S_x \leq M_p \quad (6)$$

$$F_{cr} = C_b \frac{\pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{J}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \quad (7)$$

In which  $L_b$  is the length of unbraced segment of beam (i.e., the span in the present study), and  $C_b$  is the coefficient to account for increased moment resistance of a laterally unsupported doubly symmetric beam when subject to a moment gradient. In AISC specification [1], it does not provide a specific equation for  $C_b$  that accounts for both loading conditions and load height position effects. However, it recommends the formula provided in Ziemian [12] to determine  $C_b$ . Based on the boundary and loading conditions of the present problem, coefficient  $C_b$  is determined as

$$C_b = A.B^{2y_p/h} \quad (8)$$

where  $A = 1.35$  and  $B = 1.0 - 0.18W^2 + 0.649W$  in which

$$W = \frac{\pi}{L_b} \sqrt{\frac{EI_w}{GJ}} \quad (9)$$

#### 4. MOMENT RESISTANCE BASED ON CSA S16 SPECIFICATION [2]

For the double symmetric cross-section with class 1 section and the beam is laterally unsupported, the elastic buckling moment resistance of the beam is evaluated by the following equation.

$$M_u = \frac{C_b \pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y I_w} \quad (10)$$

in which  $I_w$  is the warping torsional constant of the section,  $J$  is the St. Venant torsional constant,  $L_b$  is the length of unbraced segment of beam (i.e., the span in the present study),  $I_y$  is the flexural moment of inertia about the weak axis of the section,  $E$  and  $G$  are the modulus of elasticity and shear modulus of the steel, and  $C_b$  is the coefficient to account for increased moment resistance of a laterally unsupported doubly symmetric beam when subject to a moment gradient and it is evaluated based on Ziemian [12] as

$$C_b = A.B^{2y_p/h} \leq 2.5 \quad (11)$$

where  $A = 1.35$  and  $B = 1.0 - 0.18W^2 + 0.649W$  in which

$$W = \frac{\pi}{L_b} \sqrt{\frac{EC_w}{GJ}} \quad (12)$$

Also, the plasticized moment resistance of a wide flange steel section can be evaluated as

$$M_r = \phi M_p = \phi Z F_y \quad (13)$$

where  $Z$  is the plastic section modulus. Once  $M_u$  and  $M_p$  are known, the factored moment resistance,  $M_r$ , of the beam shall be determined as follows: When  $M_u > 0.67M_p$ :

$$M_r = 1.15\phi M_p \left[ 1 - \frac{0.28M_p}{M_u} \right] \leq \phi M_p \quad (14)$$

And when  $M_u \leq 0.67M_p$ :

$$M_r = \phi M_u \quad (15)$$

#### 5. MOMENT RESISTANCE BASED ON EUROCODE 3 SPECIFICATION [3]

For the double symmetric cross-section with class 1 section and the beam is laterally unsupported, the factored moment resistance,  $M_r$ , of the beam is determined as follows

$$M_r = \chi_{LT} \frac{Z F_y}{\gamma_{MI}} \quad (16)$$

in which  $\gamma_{MI}$  is safety factor and it is taken as 1.0 in the present study.  $\chi_{LT}$  is the reduction factor for lateral-torsional buckling and it should not be greater than 1.0. Based on Clause 6.3.2.2 [3], the reduction factor  $\chi_{LT}$  can be evaluated as

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad (17)$$

where  $\phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right]$  in which  $\alpha_{LT}$  is an imperfection factor and it is taken as 0.21 for the rolled  $W250 \times 45$  section while  $\bar{\lambda}_{LT} = \sqrt{Z F_y / M_u}$  where  $M_u$  is the elastic critical moment for lateral-torsional buckling as similarly evaluated through Eq. (18).

For a double symmetric cross-section with classes 1 and 2 sections and the beam is laterally unsupported and simply supported, the elastic buckling moment resistance of the beam based on EC 3 [3] is evaluated by the following equation.

$$M_u = C_b \frac{\pi^2 E I_y}{L_b^2} \left( \sqrt{\frac{I_w}{I_y} + \frac{L_b^2 G J}{\pi^2 E I_y}} + (C_2 y_P)^2 - C_2 y_P \right) \quad (18)$$

In Eq. (18),  $C_b$  is a coefficient to account for increased moment resistance of a laterally unsupported doubly symmetric beam when subjected to a moment gradient.  $C_2$  is a coefficient to account for the effect of load height position. For the present example where the point load is applied at midspan, Eurocode 3 provides  $C_1 = 1.348$  and  $C_2 = 0.630$ .  $y_P$  is the distance between the point of load application and the shear center (i.e., sectional centroid in the present problem), it is positive for loads acting towards the shear center from their point of application.

## 6. MOMENT RESISTANCE BASED ON VIETNAMESE SPECIFICATION [4]

In Vietnamese design specification of steel structures TCVN5575-2012 [4], the case in which point loads applied at the centroid of the section is not provided. Instead, only the moment capacities of steel beams when a point load applied on the top or bottom flange are provided. The factored moment resistance of the system (Clause 7.2.2.1 of TCVN5575-2012) is evaluated as follows

$$M_r = f \gamma_c \phi_b W_c \quad (19)$$

in which  $f$  is the factored strength of steel, taken as 350 MPa in the present study,  $\gamma_c$  is a factor of working condition and taken as 0.95 as indicated in Table 3 of the specification,  $W_c$  is the flexural section modulus and it is evaluated as  $W_c = 2I_x/h$  where  $I_x$  is the sectional moment of inertia about strong axis,  $h$  is the depth of the section. Factor  $\phi_b$  is defined in Appendix E of TCVN5575-2012 and evaluated by

$$\phi_b = \psi \frac{I_y}{I_x} \left( \frac{h}{L_b} \right)^2 \frac{E}{f} \quad (20)$$

In order to evaluate factor  $\psi$ , a factor  $\alpha$  should be evaluated as follows

$$\alpha = 1.54 \frac{J}{I_y} \left( \frac{L_b}{h} \right)^2 \quad (\text{rolled}); \quad \alpha = 8 \left( \frac{L_b t_f}{h b_f} \right)^2 \left( 1 + \frac{h t_w^3}{2 b_f t_f^3} \right) \quad (\text{welded}) \quad (21)$$

For the case in which the point load is applied on the top flange, factor  $\psi = 1.75 + 0.09\alpha$  if  $0.1 \leq \alpha \leq 40$  and  $\psi = 3.3 + 0.053\alpha - 4.5 \times 10^{-5}\alpha^2$  if  $40 \leq \alpha \leq 400$ . Also, for the case in which the point load is applied on the bottom flange, factor  $\psi = 5.05 + 0.09\alpha$  if  $0.1 \leq \alpha \leq 40$  and  $\psi = 6.6 + 0.053\alpha - 4.5 \times 10^{-5}\alpha^2$  if  $40 \leq \alpha \leq 400$ . It is noted that the units of forces and lengths are in *DaN* and *cm*. Also, the TCVN5572-2012 specification [4] only provides the solutions when the point load is applied on the top flange or bottom flange and it does not provide a solution where the point load is applied at the section centroid (at web midheight).

## 7. MOMENT RESISTANCE BASED ON A NUMERICAL STUDY

A numerical study based on ABAQUS software [19] is developed in the present study to investigate the inelastic moment resistance of the steel beam under the effects of residual stresses, initial imperfections and positions of the point load (Figs. 1a-d). Two numerical models are proposed, in which the first model is denoted as NS-FEA in which no web stiffener is applied to the beam, while the second model is denoted as S-FEA in which three web stiffeners are applied at the mid-span and two end sections of the beam (Figs. 3a,b). The addition of web stiffeners aims at reducing web distortional effects at midspan and supports. It is noted that web stiffener requirements may be more strict in a specific specification [e.g., 1-3]. Geometric nonlinearity analyses are conducted at step level (\*STEP, NLGEOM=YES).

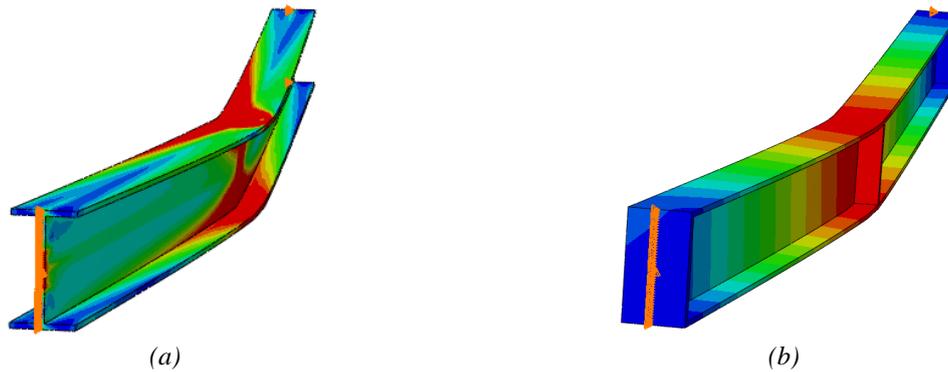


Figure 3. Two numerical models under deformation, (a) NS-FEA and (b) S-FEA

The ABAQUS models are created in .inp files (input data is written in text forms). The wide flange beams is meshed by using C3D8R elements through 5 independent numbers of elements  $n_1$  to  $n_5$  (Fig. 4) in which  $n_1$  is the number of elements across the overhang parts of the flanges,  $n_2$  is the number of elements across the flange thicknesses,  $n_3$  is the number of elements across the web thicknesses,  $n_4$  is the number of elements across the clear web depth, and  $n_5$  is the number of elements along the beam. The 8-node brick element C3D8R is selected from the ABAQUS library. The element has 24 degrees of freedom (with three translations per node) and uses reduced integration to avoid volumetric locking. Thus, the element has a single integration point located at the element centroid [20,21]. A mesh sensitivity is conducted and a mesh, with it the convergence of inelastic moment resistances are obtained, are  $n_1 = 20$ ,  $n_2 = n_3 = 4$ ,  $n_4 = 40$ ,  $n_5 = 300$  elements for span  $L = 3.0m$ ,  $n_5 = 400$  elements for span  $L = 4.0m$ , and  $n_5 = 500$  elements for span  $L = 5.0m$ .

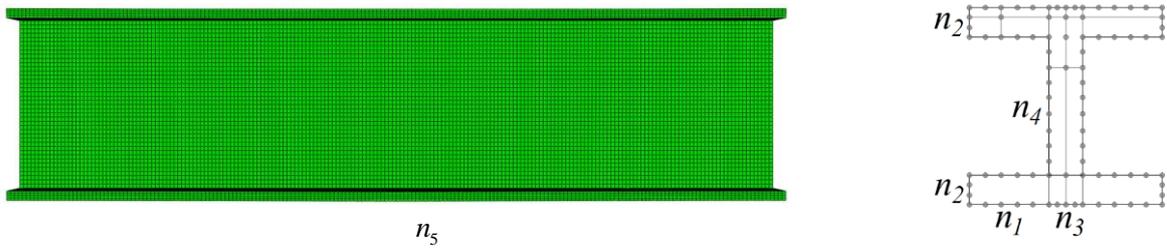


Figure 4. Five independent numbers of elements controlling the mesh of the wide flange beam

Boundary conditions of simply supports are modelled as presented in Figs. 5a,b, in which, the boundary conditions at the beam ends are applied as

```
*BOUNDARY
<Node1C0>, 2
<Node1C0>, 3
<Node1CL>, 2
U2lateralfixat0, 1
U2lateralfixatL, 1
```

where Node1C0 is the centroid of the steel cross-section at the pin end while Node1CL is that at the roller end. To avoid distortional web effects, nodes along the transversely symmetric axis of the section are restrained against lateral displacements through node set “U2lateralfixat0” and “U2lateralfixatL”. Indexes 1, 2, 3 in the \*BOUNDARY command indicate the lateral, transverse and longitudinal directions, respectively.

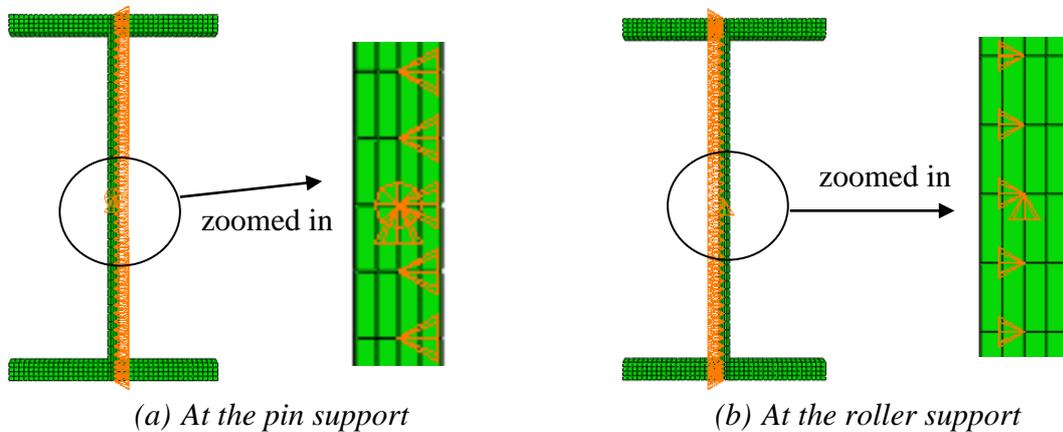


Figure 5. Boundary condition of the problem

In the S-FEA models, the thicknesses of the web stiffeners are taken as 10mm while the widths of the stiffeners are taken as the overhang parts of the flanges.

Both NS-FEA and S-FEA models include the effect of residual stresses and initial imperfections. To incorporate the effect of residual stresses, residual stress values in elements are first stored in a .csv file and they are then inputted into a .inp file by using \*INITIAL CONDITIONS, TYPE=STRESS keyword (this procedure is equivalent to a sub-routine). A blank \*STEP is finally set to balance stresses in the steel, before the loading step is evoked (Fig. 6a). The initial imperfection is incorporated into the NS-FEA and S-FEA models

through the first lateral-torsional buckling mode, which is inputted by using \*IMPERFECTION keyword (Fig. 6b). The magnitude of the initial imperfection of the beam axis is  $L/1000$  (Chapter C of AISC [1] and Clause 28.6 of CSA [2]).

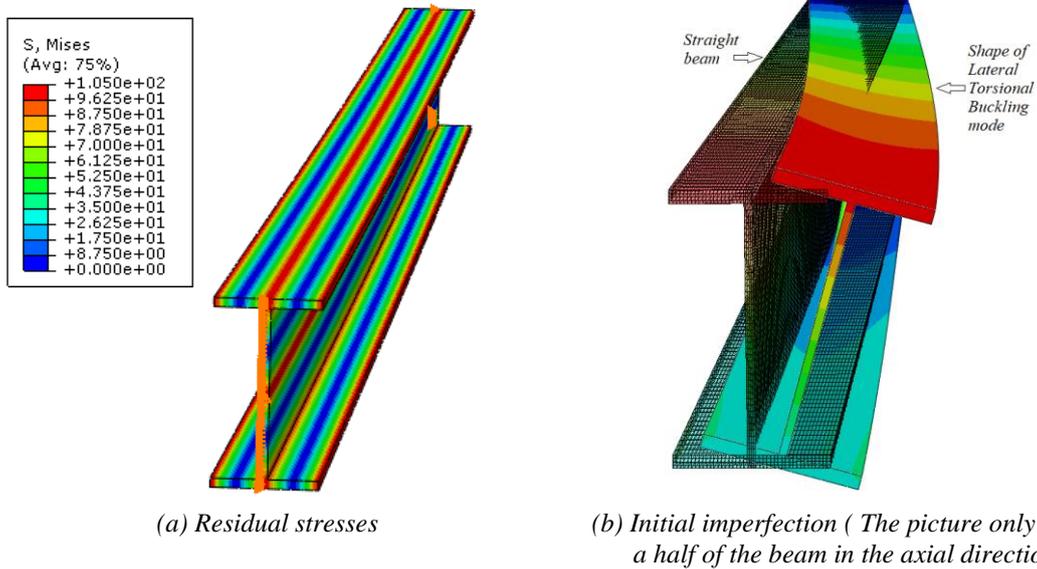
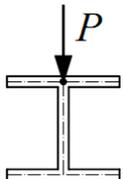


Figure 6. Implementation of residual stresses and initial imperfection in the present FEA solutions

Elastic lateral torsional buckling, inelastic buckling, and full plasticity resistances are numerically evaluated based on the NS-FEA and FEA models (e.g., Table 1 and Fig. 7 for the beam with span  $L=4m$  loaded on top flange). The moment resistances of the NS-FEA and S-FEA models are observed to be the inelastic buckling resistances (Table 1 and Fig. 7) based on local buckling effects. A description of deformation procedure of S-FEA models under loading is presented in Fig. 8.

Table 1. Moment resistances (kN.m) in the S-FEA model with span  $L=4m$

| Load position   | L=4m  |       |       |                     |
|---|-------|-------|-------|---------------------|
|   | Mu    | Mp    | Min   | S-FEA               |
|   | (1)   | (2)   | (3)   | (4) = $\min(1,2,3)$ |
|  | 161.1 | 210.7 | 129.2 | 129.2               |

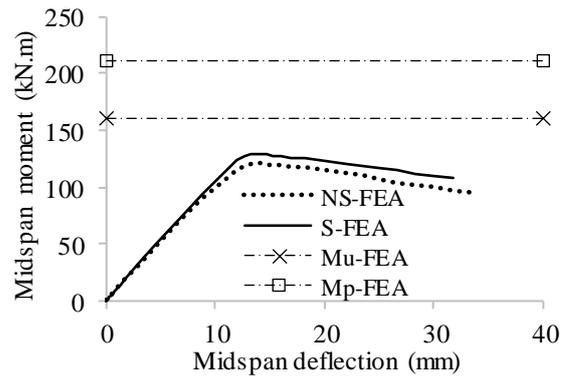


Figure 7. Midspan moment-deflection relationship for span  $L=4m$  loaded on top flange

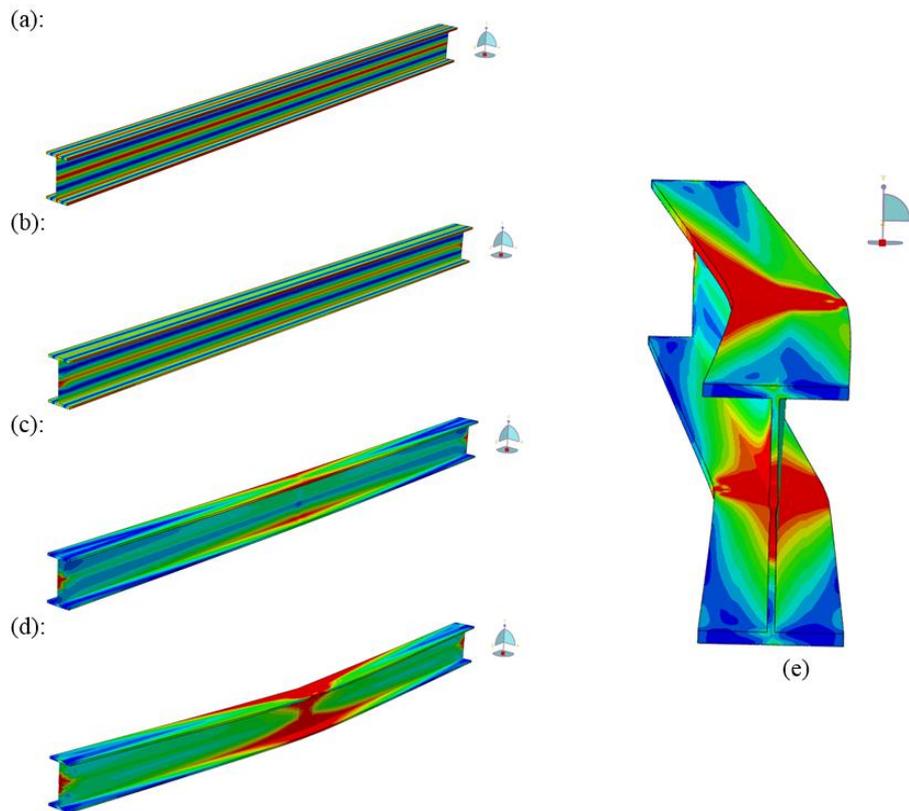


Figure 8. Responses of the steel beam under the inelastic S-FEA analysis (a) at the first step (with residual stresses, initial imperfection), (b,c) at intermediate increments, (d, e) at the final increment

## 8. RESULT DISCUSSIONS

Based on Eqs. (1) and (2) in AISC [1], limits of plasticized, inelastic buckling and elastic buckling zones can be evaluated as  $L_p = 1.48m$  and  $L_r = 4.89m$ . Based on CSA [2] is the limits are found as  $2.58m$  and  $5.90m$ , respectively. Specifications [3,4] seem not to provide the limits. This is basically a difference between the specifications. The present NS-FEA and S-FEA models are conducted for spans  $L=3, 4, 5m$  those belong to the inelastic buckling zones.

### Comparison of the inelastic buckling moment resistances predicted by specifications [1-4] and the present NS-FEA, S-FEA solutions:

Figure 9 presents the relationship between moment resistances against different unbraced spans based on specifications [1-3] and the present numerical NS-FEA and S-FEA models for the beams subjected to a point load  $P$  applied at the section centroid. Overlaid on the figure are Limits 1 and 2 based on the CSA S16 [2] specification. Among the solutions, moment resistances based on EC 3 specification are taken as a reference to compare against other solutions. Table 2 provides the values of the inelastic moment resistances. A significant difference between the specifications-based resistances can be observed, in which the AISC predicts the highest values, while the EC 3 predicts the lowest moments. For example of span  $L=4.0m$ , the moment based on EC 3 is  $147 \text{ kN.m}$ , while those based on AISC and CSA S16 are  $204.8$  and  $180.7 \text{ kN.m}$ , corresponding to the differences of  $39.3$  and  $22.9\%$ . Similar large

differences between the moments based on the AISC, CSA and EC 3 specifications can be observed for spans  $L=3.0\text{m}$  and  $L=5.0\text{m}$  (Table 2).

As observed in Fig. 9, the moment resistances based on the present numerical NS-FEA and S-FEA models lie between the EC 3 and CSA S16 solutions, in which the NS-FEA moments are relatively close to EC 3 solutions. This indicates that the present FEA solutions excellently agrees with EC 3 and CSA specifications [1,3]. The finding is consistent with the work of Kabir and Bhowmick [6]. Because there is no web stiffener in the NS-FEA model and there are 3 stiffeners in the S-FEA model, this indicates that the addition of web stiffeners increases the system capacity.

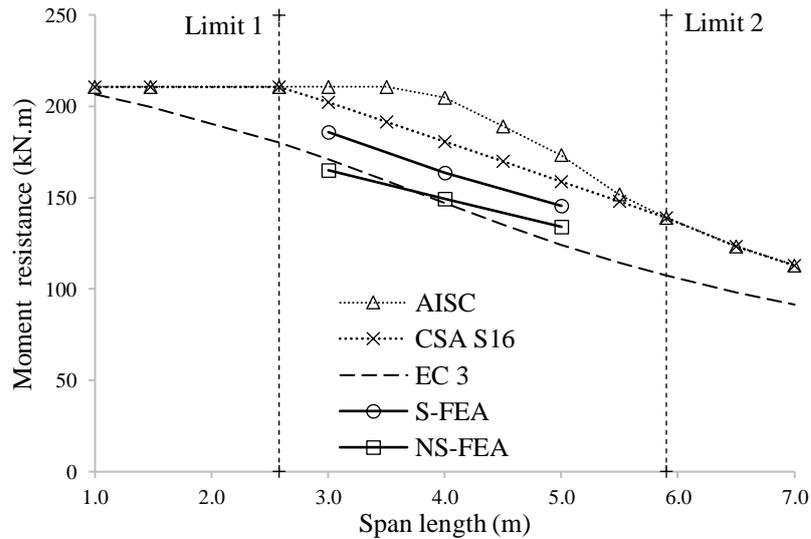


Figure 9. Moment resistance against different unbraced span lengths based on specifications [1-4] and the present numerical NS-FEA and S-FEA models for the beams loaded at the section centroid

Table 2. Inelastic moment resistances ( $kN.m$ ) and % differences based on specifications [1-4] and the present NS-FEA and S-FEA models for the beams loaded on top flange, centroid and bottom flange

| Load position    | Span (m) | Moment resistance ( $kN.m$ ) |       |         |       |        |       | % difference compared to EC 3 solution |      |         |      |        |       |
|------------------|----------|------------------------------|-------|---------|-------|--------|-------|--|------|---------|------|--------|-------|
|                  |          | EC 3                         | AISC  | CSA S16 | TCVN  | NS-FEA | S-FEA | EC 3                                   | AISC | CSA S16 | TCVN | NS-FEA | S-FEA |
| Top flange       | 3        | 145.2                        | 160.8 | 183.1   | 189.9 | 136.7  | 151.7 | 0.0                                    | 10.8 | 26.1    | 30.8 | -5.9   | 4.5   |
|                  | 4        | 117.0                        | 147.1 | 156.5   | 129.4 | 121.2  | 129.2 | 0.0                                    | 25.7 | 33.7    | 10.6 | 3.6    | 10.4  |
|                  | 5        | 97.1                         | 129.9 | 131.1   | 101.4 | 106.9  | 114.1 | 0.0                                    | 33.8 | 34.9    | 4.4  | 10.0   | 17.5  |
| Section centroid | 3        | 171.1                        | 210.7 | 202.0   | -     | 165.0  | 186.0 | 0.0                                    | 23.2 | 18.1    | -    | -3.6   | 8.7   |
|                  | 4        | 147.0                        | 204.8 | 180.7   | -     | 149.2  | 163.7 | 0.0                                    | 39.3 | 22.9    | -    | 1.5    | 11.4  |
|                  | 5        | 124.1                        | 173.2 | 158.8   | -     | 133.9  | 145.6 | 0.0                                    | 39.5 | 27.9    | -    | 7.9    | 17.3  |
| Bottom flange    | 3        | 186.3                        | 210.7 | 210.7   | 210.7 | 181.6  | 209.3 | 0.0                                    | 13.1 | 13.1    | 13.1 | -2.5   | 12.3  |
|                  | 4        | 169.5                        | 210.7 | 198.0   | 210.7 | 180.5  | 198.4 | 0.0                                    | 24.3 | 16.8    | 24.3 | 6.5    | 17.0  |
|                  | 5        | 149.6                        | 210.7 | 179.7   | 195.3 | 179.5  | 193.7 | 0.0                                    | 40.8 | 20.1    | 30.5 | 20.0   | 29.4  |

Figures 10a,b present the relationship between moment resistances against different unbraced spans based on specifications [1-4] and the present numerical NS-FEA and S-FEA models for the beams subjected to the point load  $P$  applied at the top and bottom flange,

respectively. The moment resistances based on EC 3 moments are taken as a reference to compare against other solutions. The AISC and CSA S16 solutions again overpredict the moments from 10.8 to 40.8% (Table 2).

When the load is applied at the top flange (Fig. 10a, Table 2), the moment resistance for span  $L=3.0\text{m}$  based on the TCVN is 30.8% higher than that of the EC 3. More agreements of the moments for spans greater than  $L= 4.0\text{m}$  between the TCVN and EC 3 solutions are observed in Fig. 8a. Meanwhile, the NS-FEA and S-FEA models- based moment resistances are found to well agree with the EC 3 solutions.

When the load is applied at the bottom flange (Fig. 10b, Table 2), the moment resistance for span  $L=3.0\text{m}$  based on NS-FEA agrees with that of the EC 3. However, the NS-FEA and S-FEA are generally in good agreement with the CSA S16 solutions. The moment resistances based on TCVN are found to be significantly higher than those of the EC2.

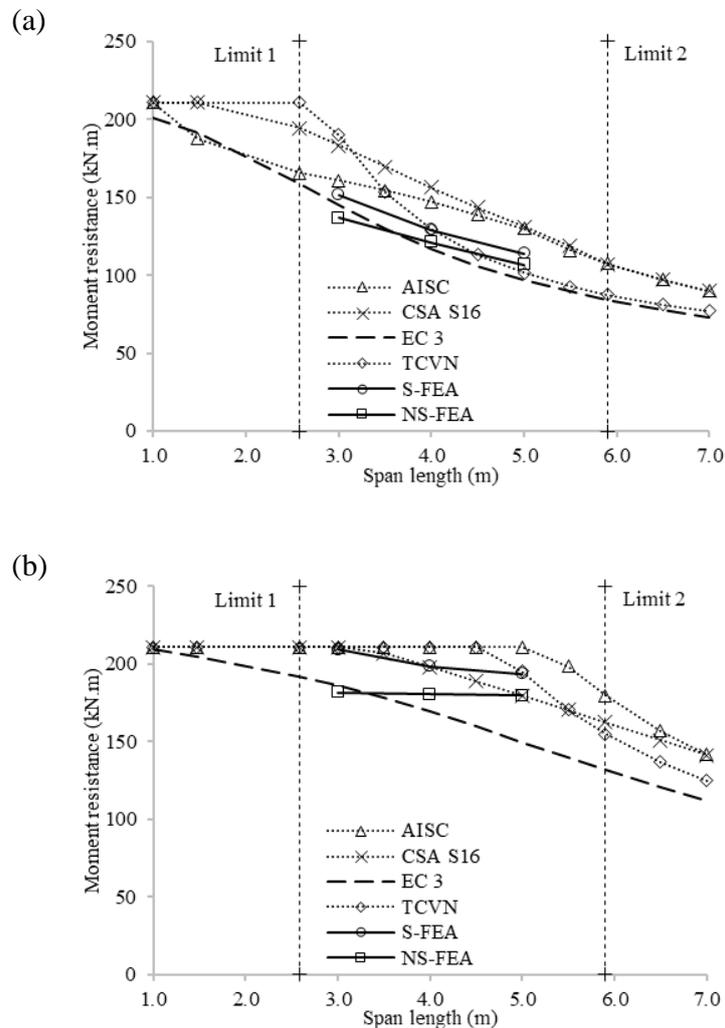


Figure 10. Moment resistance against different unbraced lengths based on specifications [1-4] and the present NS-FEA and S-FEA models for the beam loaded on the (a) top flange, (b) bottom flange

Effect of load height effects for the inelastic buckling moment resistances:

As described in Figs. 1a-d, the present study is going to investigate the effect of load positions applied on the beam cross-section. Figures 11a-c respectively present the relationship between moment resistances against different load positions for spans  $L=3.0$ ,  $4.0$  and  $5.0\text{m}$  based on the present numerical S-FEA models and the EC 3 solutions. The horizontal axis presents the position of load application, in which load position of  $-126.5\text{mm}$  corresponds to the load applied at the top flange, load position of  $0$  corresponds to the load applied at the section centroid, and load position of  $126.5\text{mm}$  corresponds to the load applied at the bottom flange. As observed, the moment resistances based on both the present and the EC 3 solutions are significantly varied according to the load position. For example of span  $L=4.0\text{m}$  (Fig. 11b), the moment based on the present S-FEA model is  $129.2\text{ kN.m}$  when the load is applied at the top flange, that is  $198.4\text{ kN.m}$  when the load is applied at the bottom flange, corresponding to a difference of upto  $53.6\%$ . Also, the moment based on the present EC 3 solution is  $117.0\text{ kN.m}$  when the load is applied at the top flange, that is  $169.5\text{ kN.m}$  when the load is applied at the bottom flange, corresponding to a difference of  $44.9\%$ . Similar observations for spans  $L=3.0$  and  $5.0\text{m}$  can be obtained.

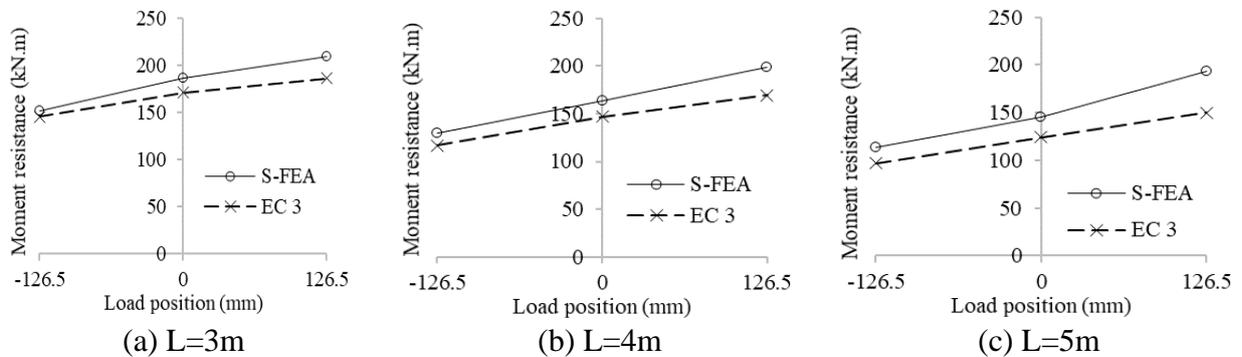


Figure 11. Moment resistance against different positions of load applications based on the present numerical NS-FEA and S-FEA models and the EC 3 solutions

## 9. CONCLUSIONS

The present study successfully developed a numerical solution in ABAQUS software to predict inelastic buckling moment resistances of rolled steel beams with compact sections and subjected to the effects of residual stresses and initial imperfections. The numerical models-based moment resistances are well-validated against four specifications AISC [1], CSA S16 [2], EC 3 [3] and TCVN5575-2012 [4]. Through comparisons between the specifications and the numerical solutions, key conclusions can be obtained in the following:

- A significant difference between the specifications-based moment resistances can be observed, in which the AISC solution predicts the highest values, while the EC 3 solution predicts the lowest moments. The difference may be explained by the different formulas used in specifications [1-4] to evaluate the moment resistances, as presented in sections 3-6 of the present study.
- The moment resistances based on the present numerical NS-FEA and S-FEA models lie between the EC 3 and CSA S16 solutions, in which the NS-FEA moments are relatively close to EC 3 solutions. The present FEA solutions excellently agree with EC 3 and CSA specifications. Also, the numerical study shows that the addition of web stiffeners increases the system capacity.

- When the load is applied at the top or bottom flange, the moment resistances based on the AISC and CSA S16 solutions are significantly higher than the EC 3 solutions. When the load is applied at the top flange, the moment resistances based on the TCVN solution are generally agree with those of the EC 3 solutions. Meanwhile, the NS-FEA and S-FEA models-based moment resistances are found to excellently agree with the EC 3 solutions. When the load is applied at the bottom flange, the moment resistances based on TCVN solution are found to be significantly higher than those of the EC 3 solution. However, the NS-FEA and S-FEA solutions are generally in good agreements with the CSA solutions.
- Effect of load height positions for the inelastic buckling moment resistances are significant, as investigated by using the present S-FEA model and the EC 3 solution. Among three positions considered (i.e., top flange, section centroid and bottom flange), the weakest moment resistances correspond to the position where the point load  $P$  is applied at the top flange.
- Among the four specifications investigated (i.e., AISC, CSA S16, EC 3, and TCVN), the EC 3 specification predicts the lowest moment resistances. Therefore, it can be considered as a conservative solution in the designing of the moment resistances.

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